39. **(a)** $P(\text{two defective chips})$

$= \dfrac{50}{10{,}000} \cdot \dfrac{49}{9{,}999} \approx 0.0000245$

(b) Assuming independence,

$P(\text{two defective chips})$

$\approx (0.005)^2$

$= 0.000025$

The difference in the results of parts (a) and (b) is only 0.0000005, so the assumption of independence did not significantly affect the probability.

40. **(a)** $P(\text{1st Roger and 2nd Rick})$

$= \dfrac{1}{6494} \cdot \dfrac{1}{6493} \approx 0.0000000237$

(b) Assuming independence,

$P(\text{1st Roger and 2nd Rick})$

$= \left(\dfrac{1}{6494}\right)^2 \approx 0.0000000237$

The results are the same to the nearest ten-billionth, so the assumption of independence did not significantly affect the probability.

41. $P(\text{45–54 yrs old}) = \dfrac{546}{2{,}160} \approx 0.253$

$P(\text{45–54 years old} \mid \text{more likely}) = \dfrac{360}{1{,}329}$

≈ 0.271

No, the events "45–54 years old" and "more likely" are not independent since the preceding probabilities are not equal.

42. The probability that a person uses social media, given they are 35–44 is:

$P(\text{social media} \mid 35\text{–}44)$

$= \dfrac{N(\text{social media and } 35\text{–}44)}{N(35\text{–}44)}$

$= \dfrac{89}{125} = 0.712$

The probability that a person uses social media is:

$P(\text{social media})$

$= \dfrac{N(\text{social media})}{N(\text{participants})}$

$= \dfrac{338}{530} \approx 0.638$

The events "social media" and "35–44 years of age" are not independent since the preceding probabilities are not equal.

43. **(a)–(d)** Answers will vary.

For part (d), of the 18 boxes in the outermost ring, 12 indicate you win if you switch while 6 indicate you lose if you switch. Assuming random selection so each box is equally likely, we get $P(\text{win if switch}) = \dfrac{12}{18} = \dfrac{2}{3} \approx 0.667$.

Section 5.5

1. permutation

2. combination

3. True

4. $n \cdot (n-1) \cdot \cdots \cdot 3 \cdot 2 \cdot 1$; $= 1$.

5. $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

6. $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

7. $10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 3{,}628{,}800$

8. $12! = 12 \cdot 11 \cdot 10 \cdot \ldots \cdot 1 = 479{,}001{,}600$

9. $0! = 1$

10. $1! = 1$

11. ${}_6P_2 = \dfrac{6!}{(6-2)!} = \dfrac{6!}{4!} = 6 \cdot 5 = 30$

12. ${}_7P_2 = \dfrac{7!}{(7-2)!} = \dfrac{7!}{5!} = 7 \cdot 6 = 42$

13. ${}_4P_4 = \dfrac{4!}{(4-4)!} = \dfrac{4!}{0!} = \dfrac{24}{1} = 24$

14. ${}_7P_7 = \dfrac{7!}{(7-7)!} = \dfrac{7!}{0!} = \dfrac{5040}{1} = 5040$

15. ${}_5P_0 = \dfrac{5!}{(5-0)!} = \dfrac{5!}{5!} = 1$

16. ${}_4P_0 = \dfrac{4!}{(4-0)!} = \dfrac{4!}{4!} = 1$

17. ${}_8P_3 = \dfrac{8!}{(8-3)!} = \dfrac{8!}{5!} = 8 \cdot 7 \cdot 6 = 336$

18. ${}_9P_4 = \dfrac{9!}{(9-4)!} = \dfrac{9!}{5!} = 9 \cdot 8 \cdot 7 \cdot 6 = 3024$

19. $_8C_3 = \dfrac{8!}{3!(8-3)!} = \dfrac{8!}{3!5!} = \dfrac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$

20. $_9C_2 = \dfrac{9!}{2!(9-2)!} = \dfrac{9!}{2!7!} = \dfrac{9 \cdot 8}{2 \cdot 1} = 36$

21. $_{10}C_2 = \dfrac{10!}{2!(10-2)!} = \dfrac{10!}{2!8!} = \dfrac{10 \cdot 9}{2 \cdot 1} = 45$

22. $_{12}C_3 = \dfrac{12!}{3!(12-3)!} = \dfrac{12!}{3!9!} = \dfrac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} = 220$

23. $_{52}C_1 = \dfrac{52!}{1!(52-1)!} = \dfrac{52!}{1!51!} = \dfrac{52}{1} = 52$

24. $_{40}C_{40} = \dfrac{40!}{40!(40-40)!} = \dfrac{40!}{40!0!} = 1$

25. $_{48}C_3 = \dfrac{48!}{3!(48-3)!} = \dfrac{48!}{3!45!} = \dfrac{48 \cdot 47 \cdot 46}{3 \cdot 2 \cdot 1}$
$= 17,296$

26. $_{30}C_4 = \dfrac{30!}{4!(30-4)!} = \dfrac{30!}{4!26!} = \dfrac{30 \cdot 29 \cdot 28 \cdot 27}{4 \cdot 3 \cdot 2 \cdot 1}$
$= 27,405$

27. *ab, ac, ad, ae, ba, bc, bd, be, ca, cb, cd, ce, da, db, dc, de, ea, eb, ec, ed*
Since there are 20 permutations, $_5P_2 = 20$.

28. *ab, ac, ad, ba, bc, bd, ca, cb, cd, da, db, dc*
Since there are 12 permutations, $_4P_2 = 12$.

29. *ab, ac, ad, ae, bc, bd, be, cd, ce, de*
Since there are 10 combinations, $_5C_2 = 10$.

30. *ab, ac, ad, bc, bd, cd*
Since there are 6 combinations, $_4C_2 = 6$.

31. Here we use the Multiplication Rule of Counting. There are six shirts and four ties, so there are $6 \cdot 4 = 24$ different shirt-and-tie combinations the man can wear.

32. Here we use the Multiplication Rule of Counting. There are five blouses and three skirts, so there are $5 \cdot 3 = 15$ different outfits that the woman can wear.

33. There are 12 ways Dan can select the first song, 11 ways to select the second song, etc. From the Multiplication Rule of Counting, there are $12 \cdot 11 \cdot \ldots \cdot 2 \cdot 1 = 12! = 479,001,600$ ways that Dan can arrange the 12 songs.

34. There are 15 ways to select the first student, 14 ways to pick the second, etc. From the Multiplication Rule of Counting, there are $15 \cdot 14 \cdot \ldots \cdot 2 \cdot 1 = 15! \approx 1.30767 \times 10^{12}$ ways that 15 students could line up.

35. There are 8 ways to pick the first city, 7 ways to pick the second, etc. From the Multiplication Rule of Counting, there are $8 \cdot 7 \cdot \ldots \cdot 2 \cdot 1 = 8! = 40,320$ different routes possible for the salesperson.

36. There are 10 ways to select the first song, 9 ways to select the second, etc. From the Multiplication Rule of Counting, there are $10 \cdot 9 \cdot \ldots \cdot 2 \cdot 1 = 10! = 3,628,800$ different ways the CD player can play the 10 songs.

37. Since the company name can be represented by 1, 2, or 3 letters, we find the total number of 1 letter abbreviations, 2 letter abbreviations, and 3 letter abbreviations, then we sum the results to obtain the total number of abbreviations possible. There are 26 letters that can be used for abbreviations, so there are 26 one-letter abbreviations. Since repetitions are allowed, there are $26 \cdot 26 = 26^2$ different two-letter abbreviations, and $26 \cdot 26 \cdot 26 = 26^3$ different three-letter abbreviations. Therefore, the maximum number of companies that can be listed on the New York Stock Exchange is
$26 \text{ (one letter)} + 26^2 \text{ (two letters)}$
$+26^3 \text{ (three letters)} = 18,278$ companies.

38. Since the company name can be represented by 4 or 5 letters, we find the total number of 4 letter abbreviations and 5 letter abbreviations, then we sum the results to obtain the total number of abbreviations possible. There are 26 letters that can be used for abbreviations and repetitions are allowed, so there are $26 \cdot 26 \cdot 26 \cdot 26 = 26^4$ different four-letter abbreviations, and $26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 = 26^5$ different five-letter abbreviations. Therefore, the maximum number of companies that can be listed on the NASDAQ is
$26^4 \text{ (four letters)} + 26^5 \text{ (five letters)} = 12,338,352$.

39. **(a)** $10 \cdot 10 \cdot 10 \cdot 10 = 10^4 = 10,000$ different codes are possible.

(b) $P(\text{guessing the correct code}) = \dfrac{1}{10,000}$
$= 0.0001$

40. **(a)** $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^9$

There are $10^9 = 1{,}000{,}000{,}000$ different Social Security numbers that can be formed.

(b) $P\begin{pmatrix}\text{guessing President's} \\ \text{Soc. Sec. number}\end{pmatrix} = \dfrac{1}{10^9}$

$= 0.000000001$

41. Since lower and uppercase letters are considered the same, each of the 8 letters to be selected has 26 possibilities. Therefore, there are $26^8 = 208{,}827{,}064{,}576$ different user names possible for the local area network.

42. Each of the first 7 characters has 26 possibilities. The last character has 26 + 10 = 36 possibilities (due to the addition of the 10 digits). There are $26^7 \cdot 36 = 289{,}145{,}166{,}336$ total user names possible under this scheme. The number of additional user names is

$26^7 \cdot 36 - 26^8$

$= 289{,}145{,}166{,}336 - 208{,}827{,}064{,}576$

$= 80{,}318{,}101{,}760$

There are $80{,}318{,}101{,}760$ additional user names available.

43. **(a)** $50 \cdot 50 \cdot 50 = 50^3 = 125{,}000$ different combinations are possible.

(b) $P(\text{guessing combination}) = \dfrac{1}{50^3} = \dfrac{1}{125{,}000}$

$= 0.000008$

44. $26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 26 \cdot 10^5 = 2{,}600{,}000$ different license plates are possible.

45. Since order matters, we use the permutation formula ${}_nP_r$.

$${}_{40}P_3 = \frac{40!}{(40-3)!} = \frac{40!}{37!} = 40 \cdot 39 \cdot 38 = 59{,}280$$

There are 59,280 ways in which the top three cars can result.

46. Since order matters, we use the permutation formula ${}_nP_r$.

$${}_{10}P_2 = \frac{10!}{(10-2)!} = \frac{10!}{8!} = 10 \cdot 9 = 90$$

In a 10-horse race, there are 90 ways in which the top two horses can result.

47. Since the order of selection determines the office of the member, order matters. Therefore, we use the permutation formula ${}_nP_r$.

$${}_{20}P_4 = \frac{20!}{(20-4)!} = \frac{20!}{16!} = 20 \cdot 19 \cdot 18 \cdot 17$$
$$= 116{,}280$$

There are 116,280 different leadership structures possible.

48. Since the order of selection determines the office of the member, order matters. Therefore, we use the permutation formula ${}_nP_r$.

$${}_{50}P_4 = \frac{50!}{(50-4)!} = \frac{50!}{46!} = 50 \cdot 49 \cdot 48 \cdot 47$$
$$= 5{,}527{,}200$$

There are 5,527,200 different leadership structures possible.

49. Since the problem states the numbers must be matched in order, this is a permutation problem.

$${}_{25}P_4 = \frac{25!}{(25-4)!} = \frac{25!}{21!} = 25 \cdot 24 \cdot 23 \cdot 22$$
$$= 303{,}600$$

There are 303,600 different outcomes possible for this game.

50. Since the order of selection doesn't matter, this is a combination problem.

$${}_{21}C_9 = \frac{21!}{9!(21-9)!} = \frac{21!}{9!12!}$$
$$= \frac{21 \cdot 20 \cdot 19 \cdot \ldots \cdot 13}{9 \cdot 8 \cdot 7 \cdot \ldots \cdot 1}$$
$$= 293{,}930$$

There are 293,930 different committee structures possible.

51. Since order of selection does not matter, this is a combination problem.

$${}_{50}C_5 = \frac{50 \cdot 49 \cdot 48 \cdot 47 \cdot 46}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2{,}118{,}760$$

There are 2,118,760 different simple random samples of size 5 possible.

52. Since order of selection does not matter, this is a combination problem.

${}_{100}C_7 \approx 1.60076 \times 10^{10}$

There are 1.60076×10^{10} different simple random samples of size 7 possible.

53. There are 6 children and we need to determine the number of ways two can be boys. The order of the two boys does not matter, so this is a combination problem. There are

$${}_6C_2 = \frac{6!}{2!(6-2)!} = \frac{6!}{2!\,4!} = \frac{6\cdot 5}{2\cdot 1} = 15 \text{ ways to}$$

select 2 of the 6 children to be boys (the rest are girls), so there are 15 different birth and gender orders possible.

54. There are 8 children and we need to determine the number of ways three can be boys. The order of the three boys does not matter, so this is a combination problem. There are

$${}_8C_3 = \frac{8!}{3!(8-3)!} = \frac{8!}{3!\,5!} = \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1} = 56 \text{ ways to}$$

select 3 of the 8 children to be boys (the rest are girls), so there are 56 different birth and gender orders possible.

55. Since there are three A's, two C's, two G's, and three T's from which to form a DNA sequence, we can make $\frac{10!}{3!\cdot 2!\cdot 2!\cdot 3!} = 25{,}200$ distinguishable DNA sequences.

56. Since there is one A, four C's, three G's, and four T's from which to form a DNA sequence, we can make $\frac{12!}{1!\cdot 4!\cdot 3!\cdot 4!} = 138{,}600$ different DNA sequences.

57. Arranging the trees involves permutations with repetitions. Using Formula (3), we find that there are $\frac{11!}{4!\cdot 5!\cdot 2!} = 6930$ different ways the landscaper can arrange the trees.

58. Creating the lineup involves a permutation with repetitions. Using Formula (3), we find that there are $\frac{9!}{3!\cdot 4!\cdot 1!\cdot 1!} = 2520$ different batting orders possible.

59. Since the order of the balls does not matter, this is a combination problem. Using Formula (2), we find there are ${}_{39}C_5 = 575{,}757$ possible choices (without regard to order), so

$$P(\text{winning}) = \frac{1}{575{,}757} \approx 0.00000174.$$

60. The order of the balls doesn't matter, so we will use combinations. However, there is also a sequence of events (first urn, then second urn), so we need the Multiplication Rule of Counting as well. There are ${}_{56}C_5 = 3{,}819{,}816$ choices from the first urn and 46 from the second, so there are (3,819,816)(46) = 175,711,536 ways to select the 6 numbers. Then,

$$P(\text{winning}) = \frac{1}{175{,}711{,}536} \approx 0.0000000057.$$

61. **(a)** $P(\text{all students}) = \frac{{}_8C_5}{{}_{18}C_5}$

$$= \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1}\cdot\frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{18\cdot 17\cdot 16\cdot 15\cdot 14}$$

$$= \frac{1}{153} \approx 0.0065$$

(b) $P(\text{all faculty}) = \frac{{}_{10}C_5}{{}_{18}C_5}$

$$= \frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{5\cdot 4\cdot 3\cdot 2\cdot 1}\cdot\frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{18\cdot 17\cdot 16\cdot 15\cdot 14}$$

$$= \frac{1}{34} \approx 0.0294$$

(c) $P(2 \text{ students and } 3 \text{ faculty}) = \frac{{}_8C_2 \cdot {}_{10}C_3}{{}_{18}C_5}$

$$= \frac{8\cdot 7}{2\cdot 1}\cdot\frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1}\cdot\frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{18\cdot 17\cdot 16\cdot 15\cdot 14}$$

$$= \frac{20}{51} \approx 0.3922$$

62. **(a)** $P(\text{all Democrats})$

$$= \frac{{}_{55}C_7}{{}_{100}C_7}$$

$$= \frac{55\cdot 54\cdot 53\cdot 52\cdot 51\cdot 50\cdot 49}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}\cdot\frac{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{100\cdot 99\cdot 98\cdot 97\cdot 96\cdot 95\cdot 94}$$

$$\approx 0.0127$$

(b) $P(\text{all Republicans})$

$$= \frac{{}_{45}C_7}{{}_{100}C_7}$$

$$= \frac{45\cdot 44\cdot 43\cdot 42\cdot 41\cdot 40\cdot 39}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}\cdot\frac{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{100\cdot 99\cdot 98\cdot 97\cdot 96\cdot 95\cdot 94}$$

$$\approx 0.0028$$

(c) $P(3 \text{ Democrats and } 4 \text{ Republicans})$

$$= \frac{{}_{55}C_3 \cdot {}_{45}C_4}{{}_{100}C_7}$$

$$\approx 0.2442$$

63. $P(\text{shipment is rejected})$

$$= 1 - P(\text{none defective})$$

$$= 1 - \frac{{}_{116}C_4}{{}_{120}C_4}$$

$$= 1 - \frac{116 \cdot 115 \cdot 114 \cdot 113}{4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{4 \cdot 3 \cdot 2 \cdot 1}{120 \cdot 119 \cdot 118 \cdot 117}$$

$$\approx 0.1283$$

There is a probability of 0.1283 that the shipment is rejected.

64. $P(\text{selecting two 60-watt bulbs}) = \frac{18}{30} \cdot \frac{17}{29} \approx 0.3517$

There is a probability of 0.3517 that two 60-watt bulbs will be selected from the box.

65. (a) $P(\text{you like 2 of the 4 songs}) = \frac{{}_{5}C_2 \cdot {}_{8}C_2}{{}_{13}C_4} \approx 0.3916$

There is a probability of 0.3916 that you will like 2 of the 4 songs played.

(b) $P(\text{you like 3 of the 4 songs}) = \frac{{}_{5}C_3 \cdot {}_{8}C_1}{{}_{13}C_4} \approx 0.1119$

There is a probability of 0.1119 that you will like 3 of the 4 songs played.

(c) $P(\text{you like all 4 songs}) = \frac{{}_{5}C_4 \cdot {}_{8}C_0}{{}_{13}C_4} \approx 0.0070$

There is a probability of 0.007 that you will like all 4 songs played.

66. (a) $P(2 \text{ of the 3 cans are diet soda}) = \frac{{}_{3}C_2 \cdot {}_{9}C_1}{{}_{12}C_3} \approx 0.1227$

There is a probability of 0.1227 that exactly two cans will contain diet soda.

(b) $P(1 \text{ of the 3 cans are diet soda}) = \frac{{}_{3}C_1 \cdot {}_{9}C_2}{{}_{12}C_3} \approx 0.4909$

There is a probability of 0.4909 that 1 of the three cans will contain diet soda.

(c) $P(\text{all 3 cans are diet soda}) = \frac{{}_{3}C_3 \cdot {}_{9}C_0}{{}_{12}C_3} \approx 0.0045$

There is a probability of 0.0045 that all 3 cans will contain diet soda.

67. (a) Five cards can be selected from a deck in ${}_{52}C_5 = 2{,}598{,}960$ ways.

(b) There are ${}_{4}C_3 = 4$ ways of choosing 3 two's, and so on for each denomination. Hence, there are $13 \cdot 4 = 52$ ways of choosing three of a kind.

(c) There are ${}_{12}C_2 = 66$ choices of two additional denominations (different from that of the three of a kind) and 4 choices of suit for the first remaining card and then, for each choice of suit for the first remaining card, there are 4 choices of suit for the last card. This gives a total of $66 \cdot 4 \cdot 4 = 1056$ ways of choosing the last two cards.

(d) $P(\text{three of a kind}) = \frac{52 \cdot 1056}{2{,}598{,}960} \approx 0.0211$

68. (a) Five cards can be selected from a deck in ${}_{52}C_5 = 2{,}598{,}960$ ways.

(b) There are ${}_{4}C_2 = 6$ ways of choosing 2 two's, and so on for each denomination. Hence, there are $13 \cdot 6 = 78$ ways of choosing a pair.

(c) There are ${}_{12}C_3 = 220$ choices of three additional denominations (different from that of the pair) and 4 choices of suit for the first remaining card and then, for each choice of suit for the first remaining card, there are 4 choices of suit for the second remaining card, and likewise for the last card. This gives a total of $220 \cdot 4 \cdot 4 \cdot 4 = 14{,}080$ ways of choosing the last three cards.

(d) $P(\text{being dealt exactly a pair}) = \frac{78 \cdot 14{,}080}{2{,}598{,}960} \approx 0.4226$

69. $P(\text{all 4 modems work}) = \frac{17}{20} \cdot \frac{16}{19} \cdot \frac{15}{18} \cdot \frac{14}{17} \approx 0.4912$

There is a probability of 0.4912 that the shipment will be accepted.