27. The probability that an individual satellite will not detect a missile is $1 - 0.9 = 0.1$, so

$$P\left(\begin{matrix}\text{none of the 4}\\ \text{will detect the missile}\end{matrix}\right) = (0.1)^4 = 0.0001.$$

Thus,

$$P\left(\begin{matrix}\text{at least one of}\\ \text{the 4 satellites}\\ \text{will detect the}\\ \text{missile}\end{matrix}\right) = 1 - 0.0001 = 0.9999.$$

Answer will vary. Generally, one would probably feel safe since only 1 in 10,000 missiles should go undetected.

28. Since, the events are independent, the probability that a randomly selected household is audited and owns a dog is:

$$\begin{aligned} &P(\text{audited and owns a dog})\\ &= P(\text{audited}) \cdot P(\text{owns a dog})\\ &= (0.0642)(0.39) \approx 0.025 \end{aligned}$$

29. Since, the events are independent, the probability that a randomly selected pregnancy will result in a girl and weight gain over 40 pounds is:

$$\begin{aligned} &P(\text{girl and weight gain over 40 pounds})\\ &= P(\text{girl}) \cdot P(\text{weight gain of 40 pounds})\\ &= (0.495)(0.201) \approx 0.099 \end{aligned}$$

30. The probability that all 3 stocks increase by 10% is:

$$\begin{aligned} &P(\text{all 3 stocks increase by } 10\%)\\ &= P(\#1 \text{ up } 10\%) \cdot P(\#2 \text{ up } 10\%) \cdot P(\#3 \text{ up } 10\%)\\ &= (0.70)(0.55)(0.20) = 0.077 \end{aligned}$$

This is not unusual, $0.077 > 0.05$.

31. **(a)** $P\left(\begin{matrix}\text{male and bets on}\\ \text{professional sports}\end{matrix}\right)$

$$\begin{aligned} &= P(\text{male}) \cdot P\left(\begin{matrix}\text{bets on}\\ \text{professional}\\ \text{sports}\end{matrix}\right)\\ &= (0.484)(0.170)\\ &\approx 0.0823 \end{aligned}$$

(b) $P(\text{male or bets on professional sports})$

$$\begin{aligned} &= P(\text{male}) + P(\text{bets}) - P(\text{male and bets})\\ &= 0.17 + 0.484 - 0.0823\\ &= 0.5717 \end{aligned}$$

(c) Since $P\left(\begin{matrix}\text{male and bets on}\\ \text{professional sports}\end{matrix}\right) = 0.106,$

but we computed it as 0.0823 assuming independence, it appears that the independence assumption is not correct.

(d) $P(\text{male or bets on professional sports})$

$$\begin{aligned} &= P(\text{male}) + P(\text{bets}) - P(\text{male and bets})\\ &= 0.17 + 0.484 - 0.106\\ &= 0.548 \end{aligned}$$

The actual probability is lower than we computed assuming independence.

32. **(a)** $P\left(\begin{matrix}\text{all 24 squares}\\ \text{filled correctly}\end{matrix}\right) = \left(\frac{1}{2}\right)^{24} \approx 5.96 \times 10^{-8}$

(b) $P\left(\begin{matrix}\text{determine complete}\\ \text{configuration}\end{matrix}\right)$

$$\begin{aligned} &= \left(\frac{1}{2}\right)^{24} \cdot \left(\frac{1}{2}\right)^{4} \cdot \left(\frac{1}{2}\right)^{8}\\ &= \left(\frac{1}{2}\right)^{36}\\ &\approx 1.46 \times 10^{-11} \end{aligned}$$

33. Assuming that gender of children for different births are independent then the fact that the mother already has three girls does not affect the likelihood of having a fourth girl.

34. The events "luggage check time" and "lost luggage" are not independent events because the likelihood of lost luggage is affected by whether Ken and Dorothy check their luggage late.

Section 5.4

1. F; E

2. No, events E and F are not independent because $P(E \mid F) \neq P(E)$.

3. $P(F \mid E) = \dfrac{P(E \text{ and } F)}{P(E)} = \dfrac{0.6}{0.8} = 0.75$

4. $P(F \mid E) = \dfrac{P(E \text{ and } F)}{P(E)} = \dfrac{0.21}{0.4} = 0.525$

5. $P(F \mid E) = \dfrac{N(E \text{ and } F)}{N(E)} = \dfrac{420}{740} = 0.568$

6. $P(F \mid E) = \dfrac{N(E \text{ and } F)}{N(E)} = \dfrac{380}{925} = 0.411$

7. $P(E \text{ and } F) = P(E) \cdot P(F \mid E)$
$= (0.8)(0.4)$
$= 0.32$

8. $P(E \text{ and } F) = P(E) \cdot P(F \mid E)$
$= (0.4)(0.6)$
$= 0.24$

9. No, the events "earn more than \$100,000 per year" and "earned a bachelor's degree" are not independent because *P*("earn more than \$100,000 per year" | "earned a bachelor's degree") ≠ *P*("earn more than \$100,000 per year").

10. No, the events "bachelor's degree" and "lives in Washington D.C." are not independent because *P*("bachelor's degree" | "lives in Washington D.C.") ≠ *P*("bachelor's degree").

11. $P(\text{club}) = \frac{13}{52} = \frac{1}{4}$;

$P(\text{club} \mid \text{black card}) = \frac{13}{26} = \frac{1}{2}$

12. $P(\text{king}) = \frac{4}{52} = \frac{1}{13}$; $P(\text{king} \mid \text{heart}) = \frac{1}{13}$; No, the knowledge that the card was a heart did not change the probability that the card was a king. This means that the events "king" and "hearts" are independent.

13. $P(\text{rainy} \mid \text{cloudy}) = \frac{P(\text{rainy and cloudy})}{P(\text{cloudy})}$

$= \frac{0.21}{0.37} \approx 0.568$

14. $P(\text{cancer death} \mid 25\text{–}34 \text{ years old})$

$= \frac{P(\text{cancer death and } 25\text{–}34 \text{ years old})}{P(25\text{–}34 \text{ years old})}$

$= \frac{0.0015}{0.0171} = \frac{5}{57} \approx 0.088$

15. $P(\text{unemployed} \mid \text{high school dropout})$

$= \frac{P(\text{umployed and high school dropout})}{P(\text{high school dropout})}$

$= \frac{0.021}{0.080} \approx 0.263$

16. $P(> \$100{,}000 \text{ per year} \mid \text{lives in Northeast})$

$= \frac{P(> \$100{,}000 \text{ per year and lives in Northeast})}{P(\text{lives in Northeast})}$

$= \frac{0.054}{0.179} \approx 0.302$

17. (a) $P(\text{age } 35\text{–}44 \mid \text{more likely})$

$= \frac{N(\text{age } 35-44 \text{ and more likely})}{N(\text{more likely})}$

$= \frac{329}{1329} \approx 0.248$

(b) $P(\text{more likely} \mid \text{age } 35\text{–}44)$

$= \frac{N(\text{more likely and age } 35-44)}{N(\text{age } 35-44)}$

$= \frac{329}{536} \approx 0.614$

(c) For 18–34 year olds, the probability that they are more likely to buy a product that is 'Made in America' is:

$P(\text{More likely} \mid 18\text{–}34 \text{ years old})$

$= \frac{238}{542} \approx 0.439$

For individuals in general, the probability is:

$P(\text{More likely}) = \frac{1329}{2160} \approx 0.615$

18–34 year olds are less likely to buy a product that is labeled 'Made in America' than individuals in general.

18. (a) $P(\text{social media} \mid 18\text{–}34)$

$= \frac{N(\text{social media and } 18\text{–}34)}{N(18\text{–}34)}$

$= \frac{117}{150} = 0.78$

(b) $P(18\text{–}34 \mid \text{social media})$

$= \frac{N(18-34 \text{ and social media})}{N(\text{social media})}$

$= \frac{117}{338} \approx 0.346$

(c) Answers may vary. The probability that a person uses social media, given they are 18–34 is:

$P(\text{social media} \mid 18\text{–}34)$

$= \dfrac{N(\text{social media and } 18\text{–}34)}{N(18\text{–}34)}$

$= \dfrac{117}{150} = 0.78$

The probability that a person uses social media is:

$P(\text{social media})$

$= \dfrac{N(\text{social media})}{N(\text{participants})}$

$= \dfrac{338}{530} \approx 0.638$

There is convincing evidence that 18–34 year olds are more likely to use social media than individuals in general.

19. **(a)** $P(\text{female} \mid \text{Sunday})$

$= \dfrac{N(\text{female and Sunday})}{N(\text{Sunday})}$

$= \dfrac{2,287}{6,430} \approx 0.356$

(b) $P(\text{Sunday} \mid \text{female})$

$= \dfrac{N(\text{Sunday and female})}{N(\text{female})}$

$= \dfrac{2,287}{13,989} \approx 0.163$

(c) $P(\text{male} \mid \text{Sunday})$

$= \dfrac{N(\text{male and Sunday})}{N(\text{Sunday})}$

$= \dfrac{4,143}{6,430} \approx 0.644$

$P(\text{male} \mid \text{Monday})$

$= \dfrac{N(\text{male and Monday})}{N(\text{Monday})}$

$= \dfrac{3,178}{4,883} \approx 0.651$

$P(\text{male} \mid \text{Tuesday})$

$= \dfrac{N(\text{male and Tuesday})}{N(\text{Tuesday})}$

$= \dfrac{3,280}{5,109} \approx 0.654$

$P(\text{male} \mid \text{Wednesday})$

$= \dfrac{N(\text{male and Wednesday})}{N(\text{Wednesday})}$

$= \dfrac{3,197}{4,926} \approx 0.649$

$P(\text{male} \mid \text{Thursday})$

$= \dfrac{N(\text{male and Thursday})}{N(\text{Thursday})}$

$= \dfrac{3,389}{5,228} \approx 0.648$

$P(\text{male} \mid \text{Friday})$

$= \dfrac{N(\text{male and Friday})}{N(\text{Friday})}$

$= \dfrac{3,975}{6,154} \approx 0.646$

$P(\text{male} \mid \text{Saturday})$

$= \dfrac{N(\text{male and Saturday})}{N(\text{Saturday})}$

$= \dfrac{4,749}{7,260} \approx 0.654$

Each of the probabilities is about the same, so there does not appear to be any day where male fatalities are more likely than other days.

20. **(a)** $P(\text{no tickets} \mid \text{text})$

$= \dfrac{N(\text{no tickets and text})}{N(\text{text})}$

$= \dfrac{85}{99} \approx 0.859$

(b) $P(\text{text} \mid \text{no tickets})$

$= \dfrac{N(\text{text and no tickets})}{N(\text{no tickets})}$

$= \dfrac{85}{169} \approx 0.503$

(c) Answers may vary. The probability that a person will get 0 tickets, given they text is:

$P(0 \text{ tickets} \mid \text{text})$

$= \dfrac{N(0 \text{ tickets and text})}{N(\text{text})}$

$= \dfrac{85}{99} \approx 0.859$

The probability that a person will have 0 tickets, given they do not text is:

$P(0 \text{ tickets} \mid \text{do not text})$

$= \dfrac{N(0 \text{ tickets and do not text})}{N(\text{do not text})}$

$= \dfrac{84}{99} \approx 0.849$

These two probabilities are fairly close. There is not convincing evidence that individuals who text while driving are less likely to be issued 0 tickets than those who do not text while driving.

21. $P(\text{both televisions work})$

$= P(\text{1st works}) \cdot P(\text{2nd works} \mid \text{1st works})$

$= \dfrac{4}{6} \cdot \dfrac{3}{5} = 0.4$

$P(\text{at least one television does not work})$

$= 1 - P(\text{both televisions work})$

$= 1 - 0.4 = 0.6$

22. $P(\text{both are women})$

$= P(\text{1st woman}) \cdot P(\text{2nd woman} \mid \text{1st woman})$

$= \dfrac{4}{7} \cdot \dfrac{3}{6} = \dfrac{2}{7} \approx 0.286$

23. **(a)** $P(\text{both kings})$

$= P(\text{first king}) \cdot P(\text{2nd king} \mid \text{first king})$

$= \dfrac{4}{52} \cdot \dfrac{3}{51} = \dfrac{1}{221} \approx 0.005$

(b) $P(\text{both kings})$

$= P(\text{first king}) \cdot P(\text{2nd king} \mid \text{first king})$

$= \dfrac{4}{52} \cdot \dfrac{4}{52} = \dfrac{1}{169} \approx 0.006$

24. **(a)** $P(\text{both clubs})$

$= P(\text{first clubs}) \cdot P(\text{2nd clubs} \mid \text{first clubs})$

$= \dfrac{13}{52} \cdot \dfrac{12}{51} = \dfrac{1}{17} \approx 0.059$

(b) $P(\text{both clubs})$

$= P(\text{first clubs}) \cdot P(\text{2nd clubs} \mid \text{first clubs})$

$= \dfrac{13}{52} \cdot \dfrac{13}{52} = \dfrac{1}{16} = 0.0625$

25. $P(\text{Dave 1st and Neta 2nd})$

$= P(\text{Dave 1st}) \cdot P(\text{Neta 2nd} \mid \text{Dave 1st})$

$= \dfrac{1}{5} \cdot \dfrac{1}{4} = \dfrac{1}{20} = 0.05$

26. $P(\text{Yolanda 1st and Lorrie 2nd})$

$= P(\text{Yolanda 1st}) \cdot P(\text{Lorrie 2nd} \mid \text{Yolanda 1st})$

$= \dfrac{1}{5} \cdot \dfrac{1}{4} = \dfrac{1}{20} = 0.05$

27. **(a)** $P(\text{like both songs})$

$= P(\text{like 1st}) \cdot P(\text{like 2nd} \mid \text{like 1st})$

$= \dfrac{5}{13} \cdot \dfrac{4}{12} = \dfrac{5}{39} \approx 0.128$

The probability is greater than 0.05. This is not a small enough probability to be considered unusual.

(b) $P(\text{dislike both songs})$

$= P(\text{dislike 1st}) \cdot P(\text{dislike 2nd} \mid \text{dislike 1st})$

$= \dfrac{8}{13} \cdot \dfrac{7}{12} = \dfrac{14}{39} \approx 0.359$

(c) Since you either like both or neither or exactly one (and these are disjoint) then the probability that you like exactly one is given by

$P(\text{like exactly one song})$

$= 1 - (P(\text{like both}) + P(\text{dislike both}))$

$= 1 - \left(\dfrac{5}{39} + \dfrac{14}{39}\right) = \dfrac{20}{39} \approx 0.513$

(d) $P(\text{like both songs})$

$= P(\text{like 1st}) \cdot P(\text{like 2nd} \mid \text{like 1st})$

$= \dfrac{5}{13} \cdot \dfrac{5}{13} = \dfrac{25}{169} \approx 0.148$

The probability is greater than 0.05. This is not a small enough probability to be considered unusual.

$P(\text{dislike both songs})$

$= P(\text{dislike 1st}) \cdot P(\text{dislike 2nd} \mid \text{dislike 1st})$

$= \dfrac{8}{13} \cdot \dfrac{8}{13} = \dfrac{64}{169} \approx 0.379$

$P(\text{like exactly one song})$

$= 1 - (P(\text{like both}) + P(\text{dislike both}))$

$= 1 - \left(\dfrac{25}{169} + \dfrac{64}{169}\right) = \dfrac{80}{169} \approx 0.473$

28. **(a)** $P(\text{both are diet soda})$

$= P(\text{1st diet}) \cdot P(\text{2nd diet} \mid \text{1st diet})$

$= \frac{3}{12} \cdot \frac{2}{11} = \frac{1}{22} \approx 0.045$

(b) $P(\text{both are regular})$

$= P(\text{1st reg.}) \cdot P(\text{2nd reg.} \mid \text{1st reg.})$

$= \frac{9}{12} \cdot \frac{8}{11} = \frac{6}{11} \approx 0.545$

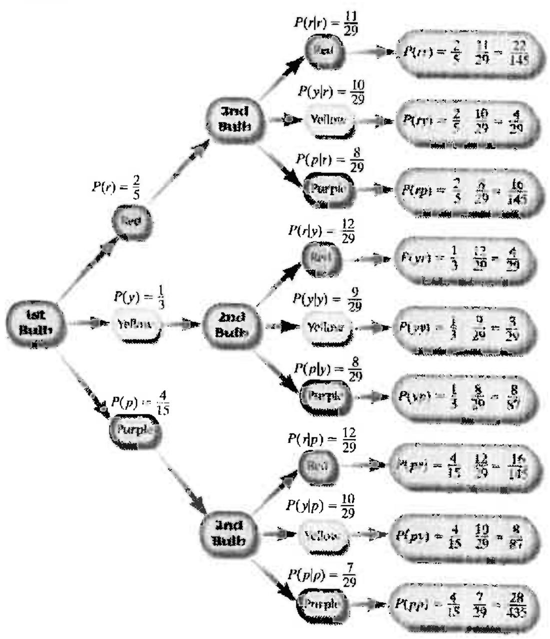
The probability is greater than 0.05. This is not a small enough probability to be unusual.

(c) Since the two cans are either both diet or both regular or one of each (and these are disjoint), the probability of one of each is $P(\text{one regular and one diet})$

$= 1 - \left(P(\text{both regular}) + P(\text{both diet})\right)$

$= 1 - \left(\frac{6}{11} + \frac{1}{22}\right) = \frac{9}{22} \approx 0.409$

29.



(a) $P(\text{both red}) = \frac{22}{145} \approx 0.152$

(b) $P(\text{1st red and 2nd yellow}) = \frac{4}{29} \approx 0.138$

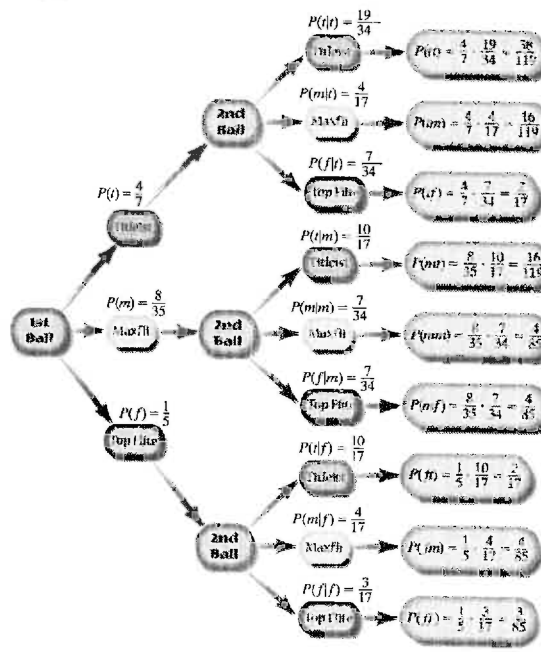
(c) $P(\text{1st yellow and 2nd red}) = \frac{4}{29} \approx 0.138$

(d) Since one each of red and yellow must be either 1st red, 2nd yellow or vice versa, by the addition rule this probability is

$P(\text{one red and one yellow}) = \frac{4}{29} + \frac{4}{29}$

$= \frac{8}{29} \approx 0.276.$

30.



(a) $P(\text{both Titleists}) = \frac{38}{119} \approx 0.319$

(b) $P(\text{1st Titleist and 2nd Maxfli}) = \frac{16}{119} \approx 0.134$

(c) $P(\text{1st Maxfli and 2nd Titleist}) = \frac{16}{119} \approx 0.134$

(d) Since either the first is a Titleist and the second a Maxfli or vice versa, this probability is

$P(\text{one Titleist and one Maxfli})$

$= \frac{16}{119} + \frac{16}{119} = \frac{32}{119} \approx 0.269$

31. $P(\text{female and smoker})$

$= P(\text{female} \mid \text{smoker}) \cdot P(\text{smoker})$

$= (0.445)(0.203) \approx 0.090$

The probability is greater than 0.05. It is not unusual to select a female who smokes.

32. $P(\text{multiple jobs and male})$

$= P(\text{male} \mid \text{multiple jobs}) \cdot P(\text{multiple jobs})$

$= (0.466)(0.049) = 0.023$

The probability is less than 0.05. It would be unusual to select a male who has multiple jobs.

33. (a) $P(\text{10 different birthdays})$

$$= \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \cdots \cdot \frac{358}{365} \cdot \frac{357}{365} \cdot \frac{356}{365}$$
$$\approx 0.883$$

(b)

$P(\text{at least 2 of the 10 people have the same birthday})$
$$= 1 - P(\text{none})$$
$$\approx 1 - 0.883$$
$$= 0.117$$

34.

$P(\text{at least 2 in a group of 23 share the same birthday})$
$$= \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \cdots \cdot \frac{343}{365}$$
$$\approx 0.493$$

$P(\text{at least 2})$
$$= 1 - P(\text{none})$$
$$\approx 1 - 0.493$$
$$= 0.507$$

35. (a) $P(\text{male}) = \frac{200}{400} = \frac{1}{2}$

$= P(\text{male} \mid \text{0 activities}) = \frac{21}{42} = \frac{1}{2}$

$P(\text{male}) = P(\text{male} \mid \text{0 activities})$, so the events "male" and "0 activities" are independent.

(b) No. $P(\text{female}) = \frac{200}{400} = \frac{1}{2} = 0.5$

$P(\text{female} \mid \text{5+ activ.}) = \frac{71}{109} \approx 0.651$

$P(\text{female}) \neq P(\text{female} \mid \text{5+ activ.})$, so the events "female" and "5+ activities" are not independent.

(c) Yes, the events "1–2 activities" and "3–4 activities" are mutually exclusive because the two events cannot occur at the same time. $P(\text{1–2 activ. and 3–4 activ.}) = 0$

(d) No, the events "male" and "1–2 activities" are not mutually exclusive because the two events can happen at the same time.

$P(\text{male and 1–2 activ.}) = \frac{81}{400}$
$$= 0.2025 \neq 0$$

36. (a) No. $P(\text{Republican}) = \frac{2200}{4000} = \frac{11}{20} = 0.55$

$P(\text{Republican} \mid \text{age 30–44}) = \frac{340}{724} = \frac{85}{181} \approx 0.470$

$P(\text{Republican}) \neq P(\text{Republican} \mid \text{age 30–44})$ so the events "Republican" and "30–44" are not independent.

(b) Yes. $P(\text{Democrat}) = \frac{1800}{4000} = \frac{9}{20} = 0.45$

$P(\text{Democrat} \mid 65+) = \frac{459}{1020} = \frac{9}{20} = 0.45$

$P(\text{Democrat}) = P(\text{Democrat} \mid 65+)$, so the events "Democrat" and "65+" are independent.

(c) Yes, the events "17–29" and "45–64" are mutually exclusive because the two events cannot happen at the same time. That is, an individual cannot be in both age groups.

(d) No, the events "Republican" and "45–64" are not mutually exclusive because the two events can happen at the same time.

$P(\text{Repub. and 45-64}) = \frac{1075}{4000} \approx 0.269 \neq 0$

37. (a) $P(\text{being dealt 5 clubs})$

$$= \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48} = \frac{33}{66{,}640}$$
$$\approx 0.000495$$

(b) $P(\text{being dealt a flush})$

$$= 4\left(\frac{33}{66{,}640}\right) = \frac{33}{16{,}660} \approx 0.002$$

38. Given that you have a flush, the probability that the five cards selected are Ten, Jack, Queen, King, Ace is

$P(\text{royal} \mid \text{flush}) = \frac{5}{13} \cdot \frac{4}{12} \cdot \frac{3}{11} \cdot \frac{2}{10} \cdot \frac{1}{9} = \frac{1}{1287}.$

Hence,

$P(\text{royal flush}) = P(\text{flush}) \cdot P(\text{royal} \mid \text{flush})$
$$= \frac{33}{16{,}660} \cdot \frac{1}{1287}$$
$$= \frac{1}{649{,}740} \approx 0.00000154$$

39. **(a)** $P(\text{two defective chips})$

$= \frac{50}{10{,}000} \cdot \frac{49}{9{,}999} \approx 0.0000245$

(b) Assuming independence,

$P(\text{two defective chips})$

$\approx (0.005)^2$

$= 0.000025$

The difference in the results of parts (a) and (b) is only 0.0000005, so the assumption of independence did not significantly affect the probability.

40. **(a)** $P(\text{1st Roger and 2nd Rick})$

$= \frac{1}{6494} \cdot \frac{1}{6493} \approx 0.0000000237$

(b) Assuming independence,

$P(\text{1st Roger and 2nd Rick})$

$= \left(\frac{1}{6494}\right)^2 \approx 0.0000000237$

The results are the same to the nearest ten-billionth, so the assumption of independence did not significantly affect the probability.

41. $P(\text{45–54 yrs old}) = \frac{546}{2{,}160} \approx 0.253$

$P(\text{45–54 years old} \mid \text{more likely}) = \frac{360}{1{,}329}$

≈ 0.271

No, the events "45–54 years old" and "more likely" are not independent since the preceding probabilities are not equal.

42. The probability that a person uses social media, given they are 35–44 is:

$P(\text{social media} \mid \text{35–44})$

$= \frac{N(\text{social media and 35–44})}{N(\text{35–44})}$

$= \frac{89}{125} = 0.712$

The probability that a person uses social media is:

$P(\text{social media})$

$= \frac{N(\text{social media})}{N(\text{participants})}$

$= \frac{338}{530} \approx 0.638$

The events "social media" and "35–44 years of age" are not independent since the preceding probabilities are not equal.

43. **(a)–(d)** Answers will vary.

For part (d), of the 18 boxes in the outermost ring, 12 indicate you win if you switch while 6 indicate you lose if you switch. Assuming random selection so each box is equally likely, we get $P(\text{win if switch}) = \frac{12}{18} = \frac{2}{3} \approx 0.667$.

Section 5.5

1. permutation
2. combination
3. True
4. $n \cdot (n-1) \cdot \dots \cdot 3 \cdot 2 \cdot 1$; $= 1$.
5. $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$
6. $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$
7. $10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 3{,}628{,}800$
8. $12! = 12 \cdot 11 \cdot 10 \cdot \ldots \cdot 1 = 479{,}001{,}600$
9. $0! = 1$
10. $1! = 1$
11. ${}_6P_2 = \frac{6!}{(6-2)!} = \frac{6!}{4!} = 6 \cdot 5 = 30$
12. ${}_7P_2 = \frac{7!}{(7-2)!} = \frac{7!}{5!} = 7 \cdot 6 = 42$
13. ${}_4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{24}{1} = 24$
14. ${}_7P_7 = \frac{7!}{(7-7)!} = \frac{7!}{0!} = \frac{5040}{1} = 5040$
15. ${}_5P_0 = \frac{5!}{(5-0)!} = \frac{5!}{5!} = 1$
16. ${}_4P_0 = \frac{4!}{(4-0)!} = \frac{4!}{4!} = 1$
17. ${}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 \cdot 7 \cdot 6 = 336$
18. ${}_9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = 9 \cdot 8 \cdot 7 \cdot 6 = 3024$