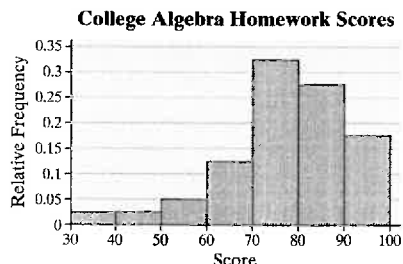


- (k) Recommendations may vary. The benefits of the decrease in red-light running crashes must be weighed against the negative of increased rear-end crashes. Seriousness of injuries and amount of property damage may need to be considered.
46. (a) Total number of homework assignments submitted = 40. One student had a score between 30 and 39. Relative frequency of "30–39" is  $1/40 = 0.025$  and so on.

Score	Relative Frequency
30–39	0.025
40–49	0.025
50–59	0.050
60–69	0.125
70–79	0.325
80–89	0.275
90–99	0.175
Total	1.000

(b)



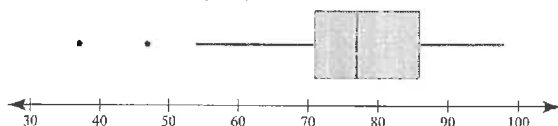
- (c) The mean is calculated by adding all the scores and dividing by the total number of scores. The mean is:

$$\bar{x} = \frac{37 + 48 + 54 + \dots + 98}{40} = \frac{3080}{40} = 77.$$

The median is the average of the value in the 20<sup>th</sup> and 21<sup>st</sup> positions:

$$\text{median} = \frac{77 + 77}{2} = 77.$$

- (d) As seen on the boxplot there are two outliers; 37, 48.



- (e) The histogram and boxplot indicate the distribution is skewed slightly to the left.

- (f) The standard deviation is 13.06 (using software). The lower quartile is the mean of the values in the 10<sup>th</sup> and 11<sup>th</sup>

$$\text{positions: } Q_1 = \frac{70 + 72}{2} = 71.$$

The upper quartile is the mean of the values in the 30<sup>th</sup> and 31<sup>st</sup> positions:

$$Q_3 = \frac{85 + 87}{2} = 86. \text{ The interquartile}$$

$$\text{range is: } Q_3 - Q_1 = 86 - 71 = 15.$$

- (g) There are 40 students, and 4 earned scores less than 60. So, the probability that a randomly selected student will have a score less than 70 is:

$$P(\text{less than } 60) = \frac{4}{40} = 0.10.$$

- (h) There are 40 students, and 18 earned scores of at least 80. So, the probability that a randomly selected student will have a score of at least 80 is:

$$P(80 \text{ or higher}) = \frac{18}{40} = 0.45.$$

- (i) There are 40 students, and 0 earned scores below 30. So, the probability that a randomly selected student will have a score below 30 is:

$$P(\text{less than } 30) = \frac{0}{40} = 0.$$

## Section 5.3

1. independent
2. Multiplication
3. Addition
4. False. Two events,  $E$  and  $F$ , are disjoint if they cannot occur simultaneously (or have no simple events in common). Disjoint events are automatically dependent, because if  $E$  occurs then  $F$  cannot occur which means the probability of  $F$  is affected by the occurrence of  $E$ .
5.  $P(E) \cdot P(F)$
6.  $P(E \text{ and } F) = 0$  since  $E$  and  $F$  cannot occur together.

7. (a) Dependent. Speeding on the interstate increases the probability of being pulled over by a police officer.  
 (b) Dependent: Eating fast food affects the probability of gaining weight.  
 (c) Independent: Your score on a statistics exam does not affect the probability that the Boston Red Sox win a baseball game.
8. (a) Independent: The state of your calculator batteries does not affect the probability that your calculator batteries are dead.  
 (b) Independent: Your choice of favorite color does not affect the probability that your friend's hobby is fishing.  
 (c) Dependent: Your car running out of gas could affect the probability that you are late for school.

9. Since  $E$  and  $F$  are independent,

$$\begin{aligned} P(E \text{ and } F) &= P(E) \cdot P(F) \\ &= (0.3)(0.6) \\ &= 0.18 \end{aligned}$$

10. Since  $E$  and  $F$  are independent,

$$\begin{aligned} P(E \text{ and } F) &= P(E) \cdot P(F) \\ &= (0.7)(0.9) \\ &= 0.63 \end{aligned}$$

$$\begin{aligned} 11. \quad P(5 \text{ heads in a row}) &= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \\ &= \left(\frac{1}{2}\right)^5 = \frac{1}{32} = 0.03125 \end{aligned}$$

If we flipped a coin five times, 100 different times, we would expect to observe 5 heads in a row about 3 times.

$$\begin{aligned} 12. \quad P(4 \text{ ones in a row}) &= \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) \\ &= \left(\frac{1}{6}\right)^4 = \frac{1}{1296} \approx 0.0008 \end{aligned}$$

This means that if a six-sided die is rolled 4 times, the result would be all ones about 0.08% of the time.

$$\begin{aligned} 13. \quad P(2 \text{ left-handed people}) &= (0.13)(0.13) \\ &= 0.0169 \end{aligned}$$

$$\begin{aligned} P(\text{At least 1 is right-handed}) &= 1 - P(2 \text{ left-handed people}) \\ &= 1 - 0.0169 = 0.9831 \end{aligned}$$

14. (a) The two lotteries are independent because the numbers are randomly drawn. Whether or not Shawn wins one of the lotteries does not affect the probability that he will win the other.

$$\begin{aligned} (b) \quad P(\text{wins both}) &= P(\text{wins MO}) \cdot P(\text{wins IL}) \\ &= (0.00000028357)(0.000000098239) \\ &\approx 0.000000000000279 \end{aligned}$$

We would expect someone to win both lotteries about 3 times in 100 trillion attempts.

$$\begin{aligned} 15. (a) \quad P(\text{all 5 negative}) &= (0.995)(0.995)(0.995)(0.995)(0.995) \\ &= (0.995)^5 \approx 0.9752 \end{aligned}$$

$$\begin{aligned} (b) \quad P(\text{at least one positive}) &= 1 - P(\text{all 5 negative}) \\ &= 1 - 0.9752 \\ &= 0.0248 \end{aligned}$$

$$\begin{aligned} 16. (a) \quad P(\text{all 100 last 2 years}) &= (0.995)^{100} \\ &\approx 0.6058 \end{aligned}$$

$$\begin{aligned} (b) \quad P(\text{at least one burns out}) &= 1 - P(\text{all 100 last 2 years}) \\ &= 1 - 0.6058 \\ &= 0.3942 \end{aligned}$$

$$\begin{aligned} 17. (a) \quad P(\text{two will live to be 41}) &= (0.99757)(0.99757) \\ &= 0.99515 \end{aligned}$$

$$\begin{aligned} (b) \quad P(5 \text{ will live to be 41}) &= (0.99757)^5 \\ &\approx 0.98791 \end{aligned}$$

- (c) This is the complement of the event in (b), so the probability is  $1 - 0.98791 = 0.01209$  which is unusual since  $0.01209 < 0.05$ .

$$\begin{aligned} 18. (a) \quad P(\text{two will live to be 41}) &= (0.99855)(0.99855) \\ &\approx 0.99710 \end{aligned}$$

$$\begin{aligned} (b) \quad P(5 \text{ will live to be 41}) &= (0.99855)^5 \\ &\approx 0.99277 \end{aligned}$$

- (c) This is the complement of the event in (b), so the probability is  $1 - 0.99277 = 0.00723$  which is unusual since  $0.00723 < 0.05$ .
19. (a) Using the complementation rule,  

$$P(\text{not default}) = 1 - P(\text{default})$$

$$= 1 - 0.01 = 0.99$$
- (b) Assuming that the likelihood of default is independent,  

$$P(5 \text{ will not default}) = (0.99)^5$$

$$= 0.951$$
- (c) Probability the derivative is worthless is the probability that at least one of the mortgages defaults,  

$$P(\text{At least 1 defaults})$$

$$= 1 - P(\text{None default})$$

$$= 1 - P(\text{All 5 will not default})$$

$$= 1 - (0.99)^5 = 1 - 0.951$$

$$= 0.049$$
- (d) The assumption that the likelihood of default is independent is probably not reasonable. Economic conditions (such as recessions) will impact all mortgages. Thus, if one mortgage defaults, the likelihood of a second mortgage defaulting may be higher.
20. (a) 
$$P\left(\begin{array}{l} \text{two inspectors} \\ \text{do not identify} \\ \text{low-quality timber} \end{array}\right) = (0.2)(0.2) = 0.04$$
- (b) From part (a), we know that two inspectors is not enough, so we check three:  

$$P\left(\begin{array}{l} \text{three inspectors} \\ \text{do not identify} \\ \text{low-quality timber} \end{array}\right) = (0.2)^3 = 0.008,$$
 which is below 1%. Thus, three inspectors should be hired in order to keep the probability of failing to identify low-quality timber less than one percent.
- (c) In repeated inspections of timbers, we expect both inspectors will fail to identify a low-quality timber about 4 times out of 100.
21. (a) Assuming each component's failure/success is independent of the others,  

$$P(\text{all three fail}) = (0.006)^3$$

$$= 0.000000216$$
- (b) At least one component not failing is the complement of all three components failing, so  

$$P(\text{at least one does not fail})$$

$$= 1 - P(\text{all 3 fail})$$

$$= 1 - (0.006)^3$$

$$= 1 - 0.00000216$$

$$= 0.99999784$$
22. (a)  $P(\text{one failure}) = 0.15$ ; this is not unusual because  $0.15 > 0.05$ . Since components fail independent of each other, we get  

$$P(\text{two failures}) = (0.15)(0.15) = 0.0225;$$
 this is unusual because  $0.0225 < 0.05$ .
- (b) This is the complement of both components failing, so  

$$P(\text{system succeeds}) = 1 - P(\text{both fail})$$

$$= 1 - (0.15)^2$$

$$= 1 - 0.0225$$

$$= 0.9775$$
- (c) From part (b) we know that two components are not enough, so we increase the number.  
 3 components:  

$$P(\text{system succeeds}) = 1 - (0.15)^3$$

$$\approx 0.99663$$
 4 components:  

$$P(\text{system succeeds}) = 1 - (0.15)^4$$

$$\approx 0.99949$$
 5 components:  

$$P(\text{system succeeds}) = 1 - (0.15)^5$$

$$\approx 0.99992$$
 Therefore, 5 components would be needed to make the probability of the system succeeding greater than 0.9999.

23. (a) At least one component not failing is the complement of all three components failing. Assuming each component's failure/success is independent of the others,

$$P(\text{system does not fail})$$

$$P(\text{at least one component works})$$

$$= 1 - P(\text{no components work})$$

$$= 1 - (0.03)^3$$

$$= 0.999973$$

- (b) From part (a) we know that three components are not enough, so we increase the number.

4 components:

$$P(\text{system succeeds}) = 1 - (0.03)^4$$

$$\approx 0.99999919$$

5 components:

$$P(\text{system succeeds}) = 1 - (0.03)^5$$

$$\approx 0.999999757$$

6 components:

$$P(\text{system succeeds}) = 1 - (0.03)^6$$

$$\approx 0.999999993$$

Therefore, 6 components would be needed to make the probability of the system succeeding greater than 0.99999999.

24. (a)  $P(\text{batter makes 10 consecutive outs}) = (0.70)^{10} \approx 0.02825$

If we randomly selected 100 different at bats of 10, we would expect about 3 to result in a streak of 10 consecutive runs.

- (b) Yes, cold streaks are unusual since the probability is  $0.02825 < 0.05$ .
- (c) The probability that the hitter makes five consecutive outs and then reaches base safely is the same as the probability that the hitter gets five outs in a row and then gets to base safely on the sixth attempt. Assuming these events are independent, it is the product of these two probabilities:

$$P(5 \text{ consecutive outs then has a base hit})$$

$$= P\left(\begin{array}{l} \text{player makes} \\ 5 \text{ consecutive} \\ \text{outs} \end{array}\right) \cdot P\left(\begin{array}{l} \text{player} \\ \text{reaches} \\ \text{base safely} \end{array}\right)$$

$$= (0.7)^5 (0.3) = 0.050421$$

- (d) Independence assumes that one at-bat doesn't affect another at-bat, which may be incorrect. If a batter isn't hitting well in recent at-bats, his or her confidence may be affected, so independence may not be a correct assumption.

25. (a)  $P\left(\begin{array}{l} \text{two strikes} \\ \text{in a row} \end{array}\right) = (0.3)(0.3) = 0.09$

(b)  $P(\text{turkey}) = (0.3)^3 = 0.027$

(c)  $P\left(\begin{array}{l} \text{gets a turkey} \\ \text{but fails to get} \\ \text{a clover} \end{array}\right) = P\left(\begin{array}{l} \text{three strikes} \\ \text{followed by} \\ \text{a non strike} \end{array}\right)$

$$= P\left(\begin{array}{l} \text{three strikes} \\ \text{in a row} \end{array}\right) \cdot P(\text{non-strike})$$

$$= (0.3)^3 (0.7) = 0.0189$$

26. (a)  $P\left(\begin{array}{l} \text{all 3 have} \\ \text{driven under} \\ \text{the influence} \\ \text{of alcohol} \end{array}\right) = (0.29)^3 \approx 0.0244$

(b)  $P\left(\begin{array}{l} \text{at least one has not driven} \\ \text{under the influence of alcohol} \end{array}\right)$

$$= 1 - P\left(\begin{array}{l} \text{all 3 have driven} \\ \text{under the influence} \\ \text{of alcohol} \end{array}\right)$$

$$= 1 - (0.29)^3$$

$$\approx 1 - 0.0244 = 0.9756$$

- (c) The probability that an individual 21- to 25-year-old has not driven while under the influence of alcohol is  $1 - 0.29 = 0.71$ , so

$$P\left(\begin{array}{l} \text{none of the} \\ 3 \text{ have driven} \\ \text{under the} \\ \text{influence of} \\ \text{alcohol} \end{array}\right) = (0.71)^3 \approx 0.3579$$

(d)  $P\left(\begin{array}{l} \text{at least one has driven under} \\ \text{the influence of alcohol} \end{array}\right)$

$$= 1 - P\left(\begin{array}{l} \text{none has driven under} \\ \text{the influence of alcohol} \end{array}\right)$$

$$= 1 - (0.71)^3$$

$$\approx 1 - 0.3579 = 0.6421$$

27. The probability that an individual satellite will not detect a missile is  $1 - 0.9 = 0.1$ , so

$$P\left(\begin{array}{c} \text{none of the 4} \\ \text{will detect the missile} \end{array}\right) = (0.1)^4 = 0.0001.$$

Thus,

$$P\left(\begin{array}{c} \text{at least one of} \\ \text{the 4 satellites} \\ \text{will detect the} \\ \text{missile} \end{array}\right) = 1 - 0.0001 = 0.9999.$$

Answer will vary. Generally, one would probably feel safe since only 1 in 10,000 missiles should go undetected.

28. Since, the events are independent, the probability that a randomly selected household is audited and owns a dog is:

$$\begin{aligned} &P(\text{audited and owns a dog}) \\ &= P(\text{audited}) \cdot P(\text{owns a dog}) \\ &= (0.0642)(0.39) \approx 0.025 \end{aligned}$$

29. Since, the events are independent, the probability that a randomly selected pregnancy will result in a girl and weight gain over 40 pounds is:

$$\begin{aligned} &P(\text{girl and weight gain over 40 pounds}) \\ &= P(\text{girl}) \cdot P(\text{weight gain of 40 pounds}) \\ &= (0.495)(0.201) \approx 0.099 \end{aligned}$$

30. The probability that all 3 stocks increase by 10% is:

$$\begin{aligned} &P(\text{all 3 stocks increase by 10\%}) \\ &= P(\#1 \text{ up } 10\%) \cdot P(\#2 \text{ up } 10\%) \cdot P(\#3 \text{ up } 10\%) \\ &= (0.70)(0.55)(0.20) = 0.077 \end{aligned}$$

This is not unusual,  $0.077 > 0.05$ .

$$\begin{aligned} 31. \text{ (a) } &P\left(\begin{array}{c} \text{male and bets on} \\ \text{professional sports} \end{array}\right) \\ &= P(\text{male}) \cdot P\left(\begin{array}{c} \text{bets on} \\ \text{professional} \\ \text{sports} \end{array}\right) \\ &= (0.484)(0.170) \\ &\approx 0.0823 \end{aligned}$$

$$\begin{aligned} \text{(b) } &P(\text{male or bets on professional sports}) \\ &= P(\text{male}) + P(\text{bets}) - P(\text{male and bets}) \\ &= 0.17 + 0.484 - 0.0823 \\ &= 0.5717 \end{aligned}$$

$$\begin{aligned} \text{(c) } &\text{Since } P\left(\begin{array}{c} \text{male and bets on} \\ \text{professional sports} \end{array}\right) = 0.106, \\ &\text{but we computed it as } 0.0823 \text{ assuming} \end{aligned}$$

independence, it appears that the independence assumption is not correct.

$$\begin{aligned} \text{(d) } &P(\text{male or bets on professional sports}) \\ &= P(\text{male}) + P(\text{bets}) - P(\text{male and bets}) \\ &= 0.17 + 0.484 - 0.106 \\ &= 0.548 \end{aligned}$$

The actual probability is lower than we computed assuming independence.

$$32. \text{ (a) } P\left(\begin{array}{c} \text{all 24 squares} \\ \text{filled correctly} \end{array}\right) = \left(\frac{1}{2}\right)^{24} \approx 5.96 \times 10^{-8}$$

$$\begin{aligned} \text{(b) } &P\left(\begin{array}{c} \text{determine complete} \\ \text{configuration} \end{array}\right) \\ &= \left(\frac{1}{2}\right)^{24} \cdot \left(\frac{1}{2}\right)^4 \cdot \left(\frac{1}{2}\right)^8 \\ &= \left(\frac{1}{2}\right)^{36} \\ &\approx 1.46 \times 10^{-11} \end{aligned}$$

33. Assuming that gender of children for different births are independent then the fact that the mother already has three girls does not affect the likelihood of having a fourth girl.

34. The events "luggage check time" and "lost luggage" are not independent events because the likelihood of lost luggage is affected by whether Ken and Dorothy check their luggage late.

## Section 5.4

1.  $F; E$

2. No, events  $E$  and  $F$  are not independent because  $P(E|F) \neq P(E)$ .

$$3. \quad P(F|E) = \frac{P(E \text{ and } F)}{P(E)} = \frac{0.6}{0.8} = 0.75$$

$$4. \quad P(F|E) = \frac{P(E \text{ and } F)}{P(E)} = \frac{0.21}{0.4} = 0.525$$

$$5. \quad P(F|E) = \frac{N(E \text{ and } F)}{N(E)} = \frac{420}{740} = 0.568$$

$$6. \quad P(F|E) = \frac{N(E \text{ and } F)}{N(E)} = \frac{380}{925} = 0.411$$