55. The Law of Large Numbers states that as the number of repetitions of a probability experiment increases (in the long term), the proportion with which a certain outcome is observed (i.e. the relative frequency) gets closer to the probability of the outcome. The games at a gambling casino are designed to benefit the casino in the long run; the risk to the casino is minimal because of the large number of gamblers.

56. Outcomes are equally likely when each outcome has the same probability of occurring.

57. An event is unusual if it has a low probability of occurring. The same "cut-off" should not always be used to identify unusual events. Selecting a "cut-off" is subjective and should take into account the consequences of incorrectly identifying an event as unusual.

58. Historically, rain has occurred on 70% of the days with otherwise similar conditions. It does not mean that it will rain 70% of the day. It could rain all day or it might not rain at all. Since the probability of rain is relatively high, it might not be a good day to plan an outdoor activity.

59. Empirical probability is based on the outcomes of a probability experiment and is the relative frequency of the event. Classical probability is based on counting techniques and is equal to the ratio of the number of ways an event can occur to the number of possible outcomes of the experiment.

60. At the beginning of the game, there is a 0.25 probability of selecting the envelope with \$100. So, the probability your envelope contains \$100 is 0.25. Once the host discards two envelopes that are known not to contain the \$100, you should switch because the probability the envelope in the host's hands has \$100 is 0.75. This is because the host knows which envelope contains the \$100, so the original probability that you did not select the \$100 envelope does not change.

61. It is impossible to be absolutely certain, but due to the law of large numbers it is most likely the smaller hospital. The larger hospital likely has more births so it less likely that the births would deviate from the expected proportion of girls.

Section 5.2

1. Two events are disjoint (mutually exclusive) if they have no outcomes in common.

2. $P(E)+P(F)$

3. $P(E)+P(F)-P(E \text{ and } F)$

4. Two events are complements when they have no outcomes in common (i.e. are disjoint) and between them contain all possible outcomes. In other words, two events E and E^c are complementary if $P(E \text{ and } E^c)=0$ and $P(E \text{ or } E^c)=1$.

5. E and $F = \{5, 6, 7\}$. No, E and F are not mutually exclusive because they have simple events in common.

6. F and $G = \{9\}$. No, F and G are not mutually exclusive because they have a simple event in common.

7. F or $G = \{5, 6, 7, 8, 9, 10, 11, 12\}$.
$P(F \text{ or } G) = P(F) + P(G) - P(F \text{ and } G)$
$= \frac{5}{12}+\frac{4}{12}-\frac{1}{12}=\frac{8}{12}=\frac{2}{3}$.

8. E or $H = \{2, 3, 4, 5, 6, 7\}$.
$P(E \text{ or } H) = P(E) + P(H) - P(E \text{ and } H)$
$= \frac{6}{12}+\frac{3}{12}-\frac{3}{12}=\frac{6}{12}=\frac{1}{2}$.

9. E and $G = \{\ \}$. Yes, E and G are mutually exclusive because they have no simple events in common.

10. F and $H = \{\ \}$. Yes, F and H are mutually exclusive because they have no simple events in common.

11. $E^c = \{1, 8, 9, 10, 11, 12\}$.
$P(E^c)=1-P(E)=1-\frac{6}{12}=\frac{1}{2}$

12. $F^c = \{1, 2, 3, 4, 10, 11, 12\}$.
$P(F^c)=1-P(F)=1-\frac{5}{12}=\frac{7}{12}$

13. $P(E \text{ or } F)=P(E)+P(F)-P(E \text{ and } F)$
$=0.25+0.45-0.15=0.55$

14. $P(E \text{ and } F)=P(E)+P(F)-P(E \text{ or } F)$
$=0.25+0.45-0.6=0.1$

15. $P(E \text{ or } F) = P(E) + P(F) = 0.25+0.45=0.7$

16. $P(E \text{ and } F) = 0$, since mutually exclusive events have no outcomes in common.

17. $P(E^c) = 1 - P(E) = 1 - 0.25 = 0.75$

18. $P(F^c) = 1 - P(F) = 1 - 0.45 = 0.55$

19. $P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$

$$0.85 = 0.60 + P(F) - 0.05$$
$$P(F) = 0.85 - 0.60 + 0.05 = 0.30$$

20. $P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$

$$0.65 = P(E) + 0.3 - 0.15$$
$$P(E) = 0.65 - 0.3 + 0.15 = 0.50$$

21. $P(\text{Titleist or Maxfli}) = \dfrac{9+8}{20} = \dfrac{17}{20} = 0.85$

22. $P(\text{Maxfli or Top-Flite}) = \dfrac{8+3}{20} = \dfrac{11}{20} = 0.55$

23. $P(\text{not Titleist}) = 1 - P(\text{Titleist})$

$$= 1 - \frac{9}{20} = \frac{11}{20} = 0.55$$

24. $P(\text{not Top-Flite}) = 1 - P(\text{Top-Flite})$

$$= 1 - \frac{3}{20} = \frac{17}{20} = 0.85$$

25. **(a)** Rule 1 is satisfied since all of the probabilities in the model are between 0 and 1.
Rule 2 is satisfied since the sum of the probabilities in the model is one: 0.472 + 0.023 + 0.025 + 0.170 + 0.122 + 0.056 + 0.132 = 1.

(b) $P(\text{rifle or shotgun}) = 0.023 + 0.025$
$= 0.048$.
If 1000 murders in 2013 were randomly selected, we would expect 48 of them to be committed with a rifle or a shotgun.

(c) $P(\text{handgun, rifle, or shotgun})$
$= 0.472 + 0.023 + 0.025$
$= 0.520$.
If 100 murders in 2013 were randomly selected, we would expect 52 of them to be committed with a handgun, a rifle, or a shotgun.

(d) To find the probability that a randomly selected murder was committed with a weapon other than a gun, we subtract the probability that a gun was used from 1.
$P(\text{not a gun}) = 1 - P(\text{a gun was used})$
$= 1 - (0.472 + 0.023 + 0.025 + 0.170)$
$= 1 - 0.690 = 0.310$
This means that there is a 31.0% probability of randomly selecting a murder that was not committed with a gun.

(e) Yes, murders with a shotgun are unusual since the probability is $0.025 < 0.05$.

26. **(a)** Rule 1 is satisfied since all of the probabilities in the model are between 0 and 1.
Rule 2 is satisfied since the sum of the probabilities in the model is one: 0.46 + 0.15 + 0.34 + 0.05 + = 1.

(b) $P\begin{pmatrix}\text{two married parents} \\ \text{in first marriage}\end{pmatrix} = 0.46$
This means that if we randomly selected 100 families with at least one child under 18 years of age, we would expect 46 of the families to have two married parents in their first marriage.

(c) $P(\text{two married parents})$
$= 0.46 + 0.15 = 0.61$
This means that if we randomly selected 100 families with at least one child under 18 years of age, we would expect 61 of the families to have two married parents.

(d) $P(\text{at least one parent}) = 1 - P(\text{no parent})$
$= 1 - 0.05 = 0.95$
This means that if we randomly selected 100 families with at least one child under 18 years of age, we would expect 95 of the families to have at least one parent at home.

27. No; for example, on one draw of a card from a standard deck, let $E = \text{diamond}$, $F = \text{club}$, and $G = \text{red card}$. Here, E and F are disjoint, as are F and G. However, E and G are *not* disjoint since diamond cards are red.

(d) No, the probability that a randomly selected worker walks to work cannot equal 0.15 because the sum of the probabilities would be more than 1 and there would be no probability model.

39. (a) Of the 137,243 men included in the study, 782 + 91 + 141 = 1014 died from cancer. Thus,

$$P(\text{died from cancer}) = \frac{1014}{137{,}243} \approx 0.007.$$

(b) Of the 137,243 men included in the study, 141 + 7725 = 7866 were current cigar smokers. Thus,

$$P(\text{current cigar smoker}) = \frac{7866}{137{,}243} \approx 0.057.$$

(c) Of the 137,243 men included in the study, 141 were current cigar smokers who died from cancer. Thus,

$$P(\text{died from cancer and current smoker}) = \frac{141}{137{,}243} \approx 0.001.$$

(d) Of the 137,243 men included in the study, 1014 died from cancer, 7866 were current cigar smokers, and 141 were current cigar smokers who died from cancer. Thus,

$$\begin{aligned} &P(\text{died from cancer or current smoker}) \\ &= P(\text{died from cancer}) + P(\text{current smoker}) \\ &\quad - P(\text{died from cancer and current smoker}) \\ &= \frac{1014}{137{,}243} + \frac{7866}{137{,}243} - \frac{141}{137{,}243} \\ &= \frac{8739}{137{,}243} \approx 0.064. \end{aligned}$$

40. (a) Of the 1700 married couples within the school boundaries, 1149 had both spouses working. Therefore,

$$P(\text{both spouses work}) = \frac{1149}{1700} \approx 0.676.$$

(b) Of the 1700 married couples within the school boundaries, 353 had 1 child under the age of 18. Therefore,

$$P(\text{1 child under 18 yrs}) = \frac{353}{1700} \approx 0.208.$$

(c) Of the 1700 married couples within the school boundaries, 370 had two or more children under the age of 18 and both spouses worked. Therefore,

$$P\begin{pmatrix}\text{both spouses work} \\ \text{and 2 or more} \\ \text{children under 18}\end{pmatrix} = \frac{370}{1700} = \frac{37}{170} \approx 0.218.$$

(d) Of the 1700 married couples within the school boundaries, 425 had only the husband working, 788 had no children under the age of 18, and 172 had only the husband working and no children under the age of 18. Therefore,

$$\begin{aligned} &P(\text{husband only or no children} < 18 \text{ yrs}) \\ &= P(\text{husband only or no children} < 18 \text{ yrs}) \\ &= P(\text{husband only}) + P(\text{no children}) \\ &\quad - P(\text{husband only and no children}) \\ &= \frac{425}{1700} + \frac{788}{1700} - \frac{172}{1700} \\ &= \frac{1041}{1700} \approx 0.612. \end{aligned}$$

(e) Of the 1700 married couples within the school boundaries, 126 had only the wife working. Therefore,

$$P(\text{only wife works}) = \frac{126}{1700} = \frac{63}{850} \approx 0.074$$

This would not be unusual since 0.074 is larger than 0.05.

41. (a) Of the 250 study participants, 100 were given the placebo. Thus,

$$P(\text{placebo}) = \frac{100}{250} = \frac{2}{5} = 0.4.$$

(b) Of the 250 study participants, 188 reported that the headache went away within 45 minutes. Thus,

$$P(\text{headache went away}) = \frac{188}{250} = \frac{94}{125} = 0.752$$

(c) Of the 250 study participants, 56 were given the placebo and reported that the headache went away. Thus,

$$P\begin{pmatrix}\text{placebo and headache} \\ \text{went away}\end{pmatrix} = \frac{56}{250} = \frac{28}{125} = 0.224$$