5. $\int (3x - 2)^4\, dx$

$u = 3x - 2,\ du = 3\, dx,\ n = 4$

Use $\int u^n\, du$.

6. $\int \frac{2t - 1}{t^2 - t + 2}\, dt$

$u = t^2 - t + 2,\ du = (2t - 1)\, dt$

Use $\int \frac{du}{u}$.

7. $\int \frac{1}{\sqrt{x}(1 - 2\sqrt{x})}\, dx$

$u = 1 - 2\sqrt{x},\ du = -\frac{1}{\sqrt{x}}\, dx$

Use $\int \frac{du}{u}$.

8. $\int \frac{2}{(2t - 1)^2 + 4}\, dt$

$u = 2t - 1,\ du = 2\, dt,\ a = 2$

Use $\int \frac{du}{u^2 + a^2}$.

9. $\int \frac{3}{\sqrt{1 - t^2}}\, dt$

$u = t,\ du = dt,\ a = 1$

Use $\int \frac{du}{\sqrt{a^2 - u^2}}$.

10. $\int \frac{-2x}{\sqrt{x^2 - 4}}\, dx$

$u = x^2 - 4,\ du = 2x\, dx,\ n = -$

Use $\int u^n\, du$.

11. $\int t \sin t^2\, dt$

$u = t^2,\ du = 2t\, dt$

Use $\int \sin u\, du$.

12. $\int \sec 3x \tan 3x\, dx$

$u = 3x,\ du = 3\, dx$

Use $\int \sec u \tan u\, du$.

13. $\int (\cos x) e^{\sin x}\, dx$

$u = \sin x,\ du = \cos x\, dx$

Use $\int e^u\, du$.

14. $\int \frac{1}{x\sqrt{x^2 - 4}}\, dx$

$u = x,\ du = dx,\ a = 2$

Use $\int \frac{du}{u\sqrt{u^2 - a^2}}$.

15. Let $u = x - 4,\ du = dx$.

$$\int 6(x - 4)^5\, dx = 6\int (x - 4)^5\, dx = 6\,\frac{(x - 4)^6}{6} + C$$
$$= (x - 4)^6 + C$$

16. Let $u = t - 9,\ du = dt$.

$$\int \frac{2}{(t - 9)^2}\, dt = 2\int (t - 9)^{-2}\, dt = \frac{-2}{t - 9} + C$$

17. Let $u = z - 4,\ du = dz$.

$$\int \frac{5}{(z - 4)^5}\, dz = 5\int (z - 4)^{-5}\, dz = 5\,\frac{(z - 4)^{-4}}{-4} + C$$
$$= \frac{-5}{4(z - 4)^4} + C$$

18. Let $u = t^3 - 1,\ du = 3t^2\, dt$.

$$\int t^2\sqrt[3]{t^3 - 1}\, dt = \frac{1}{3}\int (t^3 - 1)^{1/3}(3t^2)\, dt$$
$$= \frac{1}{3}\,\frac{(t^3 - 1)^{4/3}}{4/3} + C$$
$$= \frac{(t^3 - 1)^{4/3}}{4} + C$$

19. $$\int \left[v + \frac{1}{(3v - 1)^3}\right] dv = \int v\, dv + \frac{1}{3}\int (3v - 1)^{-3}(3)\, dv$$
$$= \frac{1}{2}v^2 - \frac{1}{6(3v - 1)^2} + C$$

20. $$\int \left[x - \frac{3}{(2x + 3)^2}\right] dx = \int x\, dx - \frac{3}{2}\int (2x + 3)^{-2}(2)\, dx$$
$$= \frac{x^2}{2} - \frac{3}{2}\,\frac{(2x + 3)^{-1}}{-1} + C$$
$$= \frac{x^2}{2} + \frac{3}{2(2x + 3)} + C$$

21. Let $u = -t^3 + 9t + 1,\ du = (-3t^2 + 9)\, dt = -3(t^2 - 3)\, dt$.

$$\int \frac{t^2 - 3}{-t^3 + 9t + 1}\, dt = -\frac{1}{3}\int \frac{-3(t^2 - 3)}{-t^3 + 9t + 1}\, dt$$
$$= -\frac{1}{3}\ln\left|-t^3 + 9t + 1\right| + C$$

22. Let $u = x^2 + 2x - 4$, $du = 2(x + 1)\,dx$.

$$\int \frac{x+1}{\sqrt{x^2+2x-4}}\,dx = \frac{1}{2}\int (x^2+2x-4)^{-1/2}(2)(x+1)\,dx$$

$$= \sqrt{x^2+2x-4} + C$$

23. $$\int \frac{x^2}{x-1}\,dx = \int (x+1)\,dx + \int \frac{1}{x-1}\,dx$$

$$= \frac{1}{2}x^2 + x + \ln|x-1| + C$$

24. $$\int \frac{2x}{x-4}\,dx = \int 2\,dx + \int \frac{8}{x-4}\,dx$$

$$= 2x + 8\ln|x-4| + C$$

25. Let $u = 1 + e^x$, $du = e^x\,dx$.

$$\int \frac{e^x}{1+e^x}\,dx = \ln(1+e^x) + C$$

26. $$\int \left(\frac{1}{3x-1} - \frac{1}{3x+1}\right)dx = \frac{1}{3}\int \frac{1}{3x-1}(3)\,dx - \frac{1}{3}\int \frac{1}{3x+1}(3)\,dx$$

$$= \frac{1}{3}\ln|3x-1| - \frac{1}{3}\ln|3x+1| + C = \frac{1}{3}\ln\left|\frac{3x-1}{3x+1}\right| + C$$

27. $$\int (1+2x^2)^2\,dx = \int (4x^4 + 4x^2 + 1)\,dx = \frac{4}{5}x^5 + \frac{4}{3}x^3 + x + C = \frac{x}{15}(12x^4 + 20x^2 + 15) + C$$

28. $$\int x\left(1 + \frac{1}{x}\right)^3 = \int x\left(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3}\right)dx = \int \left(x + 3 + \frac{3}{x} + \frac{1}{x^2}\right)dx = \frac{1}{2}x^2 + 3x + 3\ln|x| - \frac{1}{x} + C$$

29. Let $u = 2\pi x^2$, $du = 4\pi x\,dx$.

$$\int x(\cos 2\pi x^2)\,dx = \frac{1}{4\pi}\int (\cos 2\pi x^2)(4\pi x)\,dx$$

$$= \frac{1}{4\pi}\sin 2\pi x^2 + C$$

30. $$\int \sec 4x\,dx = \frac{1}{4}\int \sec(4x)(4)\,dx$$

$$= \frac{1}{4}\ln|\sec 4x + \tan 4x| + C$$

31. Let $u = \pi x$, $du = \pi\,dx$.

$$\int \csc(\pi x)\cot(\pi x)\,dx = \frac{1}{\pi}\int \csc(\pi x)\cot(\pi x)\pi\,dx$$

$$= -\frac{1}{\pi}\csc(\pi x) + C$$

32. Let $u = \cos x$, $du = -\sin x\,dx$.

$$\int \frac{\sin x}{\sqrt{\cos x}}\,dx = -\int (\cos x)^{-1/2}(-\sin x)\,dx$$

$$= -2\sqrt{\cos x} + C$$

33. Let $u = 5x$, $du = 5\,dx$.

$$\int e^{5x}\,dx = \frac{1}{5}\int e^{5x}(5)\,dx = \frac{1}{5}e^{5x} + C$$

34. Let $u = \cot x$, $du = -\csc^2 x\,dx$.

$$\int \csc^2 x e^{\cot x}\,dx = -\int e^{\cot x}(-\csc^2 x)\,dx = -e^{\cot x} + C$$

35. Let $u = 1 + e^x$, $du = e^x\,dx$.

$$\int \frac{2}{e^{-x}+1}\,dx = 2\int \left(\frac{1}{e^{-x}+1}\right)\left(\frac{e^x}{e^x}\right)dx$$

$$= 2\int \frac{e^x}{1+e^x}\,dx$$

$$= 2\ln(1+e^x) + C$$

36. $$\int \frac{5}{3e^x - 2}\,dx = 5\int \left(\frac{1}{3e^x-2}\right)\left(\frac{e^{-x}}{e^{-x}}\right)dx$$

$$= 5\int \frac{e^{-x}}{3-2e^{-x}}\,dx$$

$$= \frac{5}{2}\int \frac{1}{3-2e^{-x}}(2e^{-x})\,dx$$

$$= \frac{5}{2}\ln|3 - 2e^{-x}| + C$$

37. $\displaystyle\int \frac{\ln x^2}{x}\,dx = 2\int (\ln x)\frac{1}{x}\,dx = 2\frac{(\ln x)^2}{2} + C = (\ln x)^2 + C$

38. Let $u = \ln(\cos x)$, $du = \dfrac{-\sin x}{\cos x}\,dx = -\tan x\,dx$.

$$\int (\tan x)(\ln \cos x)\,dx = -\int (\ln \cos x)(-\tan x)\,dx$$

$$= \frac{-[\ln(\cos x)]^2}{2} + C$$

39.
$$\int \frac{1 + \sin x}{\cos x}\,dx = \int \frac{1 + \sin x}{\cos x} \cdot \frac{1 - \sin x}{1 - \sin x}\,dx$$

$$= \int \frac{1 - \sin^2 x}{\cos x(1 - \sin x)}\,dx$$

$$= \int \frac{\cos^2 x}{\cos x(1 - \sin x)}\,dx$$

$$= -\int \frac{-\cos x}{1 - \sin x}\,dx$$

$$= -\ln|1 - \sin x| + C, \quad (u = 1 - \sin x)$$

Alternate Solution:

$$\int \frac{1 + \sin x}{\cos x}\,dx = \int (\sec x + \tan x)\,dx$$

$$= \ln|\sec x + \tan x| + \ln|\sec x| + C$$

$$= \ln|\sec x(\sec x + \tan x)| + C$$

40.
$$\int \frac{1 + \cos \alpha}{\sin \alpha}\,d\alpha = \int \csc \alpha\,d\alpha + \int \cot \alpha\,d\alpha$$

$$= -\ln|\csc \alpha + \cot \alpha| + \ln|\sin \alpha| + C$$

41.
$$\frac{1}{\cos\theta - 1} = \frac{1}{\cos\theta - 1} \cdot \frac{\cos\theta + 1}{\cos\theta + 1} = \frac{\cos\theta + 1}{\cos^2\theta - 1}$$

$$= \frac{\cos\theta + 1}{-\sin^2\theta} = -\csc\theta \cdot \cot\theta - \csc^2\theta$$

$$\int \frac{1}{\cos\theta - 1}\,d\theta = \int (-\csc\theta\cot\theta - \csc^2\theta)\,d\theta$$

$$= \csc\theta + \cot\theta + C$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} + C$$

$$= \frac{1 + \cos\theta}{\sin\theta} + C$$

42.
$$\int \frac{2}{3(\sec x - 1)}\,dx = \frac{2}{3}\int \frac{1}{\sec x - 1} \cdot \left(\frac{\sec x + 1}{\sec x + 1}\right)dx$$

$$= \frac{2}{3}\int \frac{\sec x + 1}{\tan^2 x}\,dx$$

$$= \frac{2}{3}\int \frac{\sec x}{\tan^2 x}\,dx + \frac{2}{3}\int \cot^2 x\,dx$$

$$= \frac{2}{3}\int \frac{\cos x}{\sin^2 x}\,dx + \frac{2}{3}\int (\csc^2 x - 1)\,dx$$

$$= \frac{2}{3}\left(-\frac{1}{\sin x}\right) - \frac{2}{3}\cot x - \frac{2}{3}x + C$$

$$= -\frac{2}{3}[\csc x + \cot x + x] + C$$

43. Let $u = 2t - 1$, $du = 2\,dt$.

$$\int \frac{-1}{\sqrt{1 - (2t - 1)^2}}\,dt = -\frac{1}{2}\int \frac{2}{\sqrt{1 - (2t - 1)^2}}\,dt$$

$$= -\frac{1}{2}\arcsin(2t - 1) + C$$

44. Let $u = \sqrt{3}x$, $du = \sqrt{3}\,dx$.

$$\int \frac{1}{4 + 3x^2}\,dx = \frac{1}{\sqrt{3}}\int \frac{\sqrt{3}}{4 + (\sqrt{3}x)^2}\,dx$$

$$= \frac{1}{2\sqrt{3}}\arctan\left(\frac{\sqrt{3}x}{2}\right) + C$$

45. Let $u = \cos\left(\dfrac{2}{t}\right)$, $du = \dfrac{2\sin(2/t)}{t^2}\,dt$.

$$\int \frac{\tan(2/t)}{t^2}\,dt = \frac{1}{2}\int \frac{1}{\cos(2/t)}\left[\frac{2\sin(2/t)}{t^2}\right]dt$$

$$= \frac{1}{2}\ln\left|\cos\left(\frac{2}{t}\right)\right| + C$$

46. Let $u = \dfrac{1}{t}$, $du = \dfrac{-1}{t^2}\,dt$.

$$\int \frac{e^{1/t}}{t^2}\,dt = -\int e^{1/t}\left(\frac{-1}{t^2}\right)dt = -e^{1/t} + C$$

47. $\displaystyle\int \frac{3}{\sqrt{6x-x^2}}\,dx = 3\int \frac{1}{\sqrt{9-(x-3)^2}}\,dx = 3\arcsin\left(\frac{x-3}{3}\right) + C$

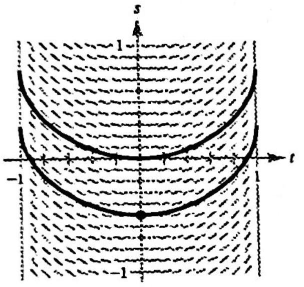
48. $\displaystyle\int \frac{1}{(x-1)\sqrt{4x^2-8x+3}}\,dx = \int \frac{2}{[2(x-1)]\sqrt{[2(x-1)]^2-1}}\,dx = \operatorname{arcsec}|2(x-1)| + C$

49. $\displaystyle\int \frac{4}{4x^2+4x+65}\,dx = \int \frac{1}{[x+(1/2)]^2+16}\,dx = \frac{1}{4}\arctan\left[\frac{x+(1/2)}{4}\right] + C = \frac{1}{4}\arctan\left(\frac{2x+1}{8}\right) + C$

50. $\displaystyle\int \frac{1}{\sqrt{1-4x-x^2}}\,dx = \int \frac{1}{\sqrt{5-(x^2+4x+4)}}\,dx = \int \frac{1}{\sqrt{5-(x+2)^2}}\,dx = \arcsin\left(\frac{x+2}{\sqrt{5}}\right) + C, \quad (a=\sqrt{5})$

51. $\displaystyle\frac{ds}{dt} = \frac{t}{\sqrt{1-t^4}}, \quad \left(0, -\frac{1}{2}\right)$

(a)



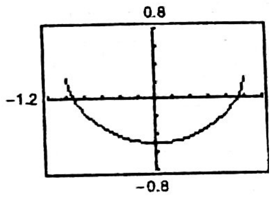
(b) $u = t^2,\ du = 2t\,dt$

$$\int \frac{t}{\sqrt{1-t^4}}\,dt = \frac{1}{2}\int \frac{2t}{\sqrt{1-(t^2)^2}}\,dt$$

$$= \frac{1}{2}\arcsin t^2 + C$$

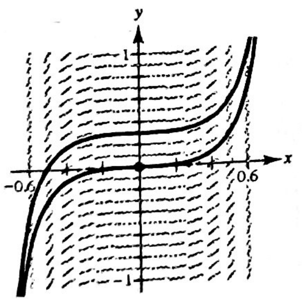
$$\left(0, -\frac{1}{2}\right):\ -\frac{1}{2} = \frac{1}{2}\arcsin 0 + C \Rightarrow C = -\frac{1}{2}$$

$$s = \frac{1}{2}\arcsin t^2 - \frac{1}{2}$$



52. $\displaystyle\frac{dy}{dx} = \tan^2(2x), \quad (0, 0)$

(a)

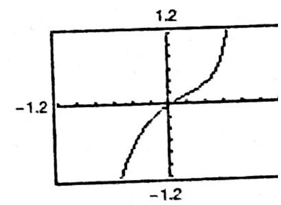


(b) $\displaystyle\int \tan^2(2x)\,dx = \int (\sec^2(2x) - 1)\,dx$

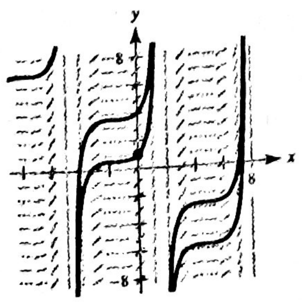
$$= \frac{1}{2}\tan(2x) - x + C$$

$$(0, 0):\ 0 = C$$

$$y = \frac{1}{2}\tan(2x) - x$$



53. (a)



(b) $\displaystyle y = \int (\sec x + \tan x)^2\,dx$

$$= \int (\sec^2 x + 2\sec x\tan x + \tan^2 x)\,dx$$

$$= \int (\sec^2 x + 2\sec x\tan x + (\sec^2 x - 1))\,dx$$

$$= \int (2\sec^2 x + 2\sec x\tan x - 1)\,dx$$

$$= 2\tan x + 2\sec x - x + C$$

At $(0, 1)$: $1 = 0 + 2 - 0 + C \Rightarrow C = -1$