1. binomial coefficients

2. Binomial Theorem *i* Pascal's Triangle

3. $\binom{n}{r}$ or ${}_nC_r$

4. expanding; binomial

21. $$(a+6)^4 = {}_4C_0a^4 + {}_4C_1a^3(6) + {}_4C_2a^2(6)^2 + {}_4C_3a(6)^3 + {}_4C_4(6)^4$$
$$= 1a^4 + 4a^3(6) + 6a^2(6)^2 + 4a(6)^3 + 1(6)^4$$
$$= a^4 + 24a^3 + 216a^2 + 864a + 1296$$

24. $$(y-2)^5 = {}_5C_0y^5 - {}_5C_1y^4(2) + {}_5C_2y^3(2)^2 - {}_5C_3y^2(2)^3 + {}_5C_4y(2)^4 - {}_5C_5(2)^5$$
$$= y^5 - 10y^4 + 40y^3 - 80y^2 + 80y - 32$$

27. $$(2x+y)^3 = {}_3C_0(2x)^3 + {}_3C_1(2x)^2(y) + {}_3C_2(2x)(y^2) + {}_3C_3(y^3)$$
$$= (1)(8x^3) + (3)(4x^2)(y) + (3)(2x)(y^2) + (1)(y^3)$$
$$= 8x^3 + 12x^2y + 6xy^2 + y^3$$

30. $$(x+2y)^4 = {}_4C_0x^4 + {}_4C_1x^3(2y) + {}_4C_2x^2(2y)^2 + {}_4C_3x(2y)^3 + {}_4C_4(2y)^4$$
$$= x^4 + 4x^3(2y) + 6x^2(4y^2) + 4x(8y^3) + 16y^4$$
$$= x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$$

33. $$(x^2+y^2)^4 = {}_4C_0(x^2)^4 + {}_4C_1(x^2)^3(y^2) + {}_4C_2(x^2)^2(y^2)^2 + {}_4C_3(x^2)(y^2)^3 + {}_4C_4(y^2)^4$$
$$= (1)(x^8) + (4)(x^6y^2) + (6)(x^4y^4) + (4)(x^2y^6) + (1)(y^8)$$
$$= x^8 + 4x^6y^2 + 6x^4y^4 + 4x^2y^6 + y^8$$

36. $$\left(\frac{1}{x}+2y\right)^6 = {}_6C_0\left(\frac{1}{x}\right)^6 + {}_6C_1\left(\frac{1}{x}\right)^5(2y) + {}_6C_2\left(\frac{1}{x}\right)^4(2y)^2 + {}_6C_3\left(\frac{1}{x}\right)^3(2y)^3 + {}_6C_4\left(\frac{1}{x}\right)^2(2y)^4 + {}_6C_5\left(\frac{1}{x}\right)(2y)^5 + {}_6C_6(2y)^6$$
$$= 1\left(\frac{1}{x}\right)^6 + 6(2)\left(\frac{1}{x}\right)^5y + 15(4)\left(\frac{1}{x}\right)^4y^2 + 20(8)\left(\frac{1}{x}\right)^3y^3 + 15(16)\left(\frac{1}{x}\right)^2y^4 + 6(32)\left(\frac{1}{x}\right)y^5 + 1(64)y^6$$
$$= \frac{1}{x^6} + \frac{12y}{x^5} + \frac{60y^2}{x^4} + \frac{160y^3}{x^3} + \frac{240y^4}{x^2} + \frac{192y^5}{x} + 64y^6$$

39. $$2(x-3)^4 + 5(x-3)^2 = 2\left[x^4 - 4(x^3)(3) + 6(x^2)(3^2) - 4(x)(3^3) + 3^4\right] + 5\left[x^2 - 2(x)(3) + 3^2\right]$$
$$= 2(x^4 - 12x^3 + 54x^2 - 108x + 81) + 5(x^2 - 6x + 9)$$
$$= 2x^4 - 24x^3 + 113x^2 - 246x + 207$$

42. 4th Row of Pascal's Triangle: 1 4 6 4 1

$$\begin{aligned}(3 - 2z)^4 &= 3^4 - 4(3)^3(2z) + 6(3)^2(2z)^2 - 4(3)(2z)^3 + (2z)^4\\ &= 81 - 216z + 216z^2 - 96z^3 + 16z^4\end{aligned}$$

45. The 4th term in the expansion of $(x + y)^{10}$ is

${}_{10}C_3 x^{10-3} y^3 = 120x^7y^3.$

48. The 4th term in the expansion of $(x - 10z)^7$ is

${}_7C_3 x^{7-3}(-10z)^3 = 35 \cdot x^4(-1000z^3) = -35{,}000x^4z^3.$

51. The 10th term in the expansion of $(10x - 3y)^{12}$ is

$$\begin{aligned}{}_{12}C_9(10x)^{12-9}(-3y)^9 &= 220(1000x^3)(-19{,}683y^9)\\ &= -4{,}330{,}260{,}000x^3y^9.\end{aligned}$$

54. The term involving x^8 in the expansion of $(x^2 + 3)^{12}$ is

$${}_{12}C_8(x^2)^4(3)^8 = \frac{12!}{(12 - 8)!8!} \cdot 3^8x^8 = 3{,}247{,}695x^8.$$

The coefficient is 3,247.695.

57. The term involving x^4y^5 in the expansion of $(2x - 5x)^9$

$$\begin{aligned}\text{is } {}_9C_5(2x)^4(-5y)^5 &= 126(16x^4)(-3125y^5)\\ &= -6{,}300{,}000x^4y^5.\end{aligned}$$

The coefficient is $-6{,}300{,}000$.

67.
$$\begin{aligned}\frac{f(x + h) - f(x)}{h} &= \frac{(x + h)^3 - x^3}{h}\\ &= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}\\ &= \frac{h(3x^2 + 3xh + h^2)}{h}\\ &= 3x^2 + 3xh + h^2, h \neq 0\end{aligned}$$

69. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{(x+h)^6 - x^6}{h}$

$= \dfrac{x^6 + 6x^5h + 15x^4h^2 + 20x^3h^3 + 15x^2h^4 + 6xh^5 + h^6 - x^6}{h}$

$= \dfrac{h\left(6x^5 + 15x^4h + 20x^3h^2 + 15x^2h^3 + 6xh^4 + h^5\right)}{h}$

$= 6x^5 + 15x^4h + 20x^3h^2 + 15x^2h^3 + 6xh^4 + h^5.\ h \neq 0$

85. ${}_7C_4\left(\dfrac{1}{2}\right)^4\left(\dfrac{1}{2}\right)^3 = \dfrac{7!}{3!4!}\left(\dfrac{1}{16}\right)\left(\dfrac{1}{8}\right) = 35\left(\dfrac{1}{16}\right)\left(\dfrac{1}{8}\right) \approx 0.273$

86. ${}_{10}C_3\left(\dfrac{1}{4}\right)^3\left(\dfrac{3}{4}\right)^7 = \dfrac{10!}{7!3!}\left(\dfrac{1}{64}\right)\left(\dfrac{2187}{16{,}384}\right) = 120\left(\dfrac{1}{64}\right)\left(\dfrac{2187}{16{,}384}\right) \approx 0.250$

87. ${}_8C_4\left(\dfrac{1}{3}\right)^4\left(\dfrac{2}{3}\right)^4 = \dfrac{8!}{4!4!}\left(\dfrac{1}{81}\right)\left(\dfrac{16}{81}\right) = 70\left(\dfrac{1}{81}\right)\left(\dfrac{16}{81}\right) \approx 0.171$

88. ${}_8C_4\left(\dfrac{1}{2}\right)^4\left(\dfrac{1}{2}\right)^4 = \dfrac{8!}{4!4!}\left(\dfrac{1}{16}\right)\left(\dfrac{1}{16}\right) = 70\left(\dfrac{1}{16}\right)\left(\dfrac{1}{16}\right) \approx 0.273$