1. mathematical induction

2. first

3. arithmetic

4. second

5. $P_k = \dfrac{5}{k(k+1)}$

$$P_{k+1} = \frac{5}{(k+1)\left[(k+1)+1\right]} = \frac{5}{(k+1)(k+2)}$$

7. $P_k = \dfrac{k^2(k+3)^2}{6}$

$$P_{k+1} = \frac{(k+1)^2\left[(k+1)+3\right]^2}{6} = \frac{(k+1)^2(k+4)^2}{6}$$

9. $P_k = \dfrac{3}{(k+2)(k+3)}$

$$P_{k+1} = \frac{3}{\left[(k+1)+2\right]\left[(k+1)+3\right]} = \frac{3}{(k+3)(k+4)}$$

11. 1. When $n = 1$, $S_1 = 2 = 1(1+1)$.

2. Assume that

$$S_k = 2 + 4 + 6 + 8 + \cdots + 2k = k(k+1).$$

Then,

$$\begin{aligned} S_{k+1} &= 2 + 4 + 6 + 8 + \cdots + 2k + 2(k+1) \\ &= S_k + 2(k+1) = k(k+1) + 2(k+1) = (k+1)(k+2). \end{aligned}$$

So, we conclude that the formula is valid for all positive integer values of n.

13. 1. When $n = 1$, $S_1 = 2 = \frac{1}{2}(5(1) - 1)$.

2. Assume that

$$S_k = 2 + 7 + 12 + 17 + \cdots + (5k - 3) = \frac{k}{2}(5k - 1).$$

Then,

$$\begin{aligned} S_{k+1} &= 2 + 7 + 12 + 17 + \cdots + (5k - 3) + \left[5(k + 1) - 3\right] \\ &= S_k + (5k + 5 - 3) = \frac{k}{2}(5k - 1) + 5k + 2 \\ &= \frac{5k^2 - k + 10k + 4}{2} = \frac{5k^2 + 9k + 4}{2} \\ &= \frac{(k + 1)(5k + 4)}{2} = \frac{(k + 1)}{2}\left[5(k + 1) - 1\right]. \end{aligned}$$

So, we conclude that this formula is valid for all positive integer values of n.

15. 1. When $n = 1$, $S_1 = 1 = 2^1 - 1$.

2. Assume that

$$S_k = 1 + 2 + 2^2 + 2^3 + \cdots + 2^{k-1} = 2^k - 1.$$

Then,

$$S_{k+1} = 1 + 2 + 2^2 + 2^3 + \cdots + 2^{k-1} + 2^k = S_k + 2^k = 2^k - 1 + 2^k = 2\left(2^k\right) - 1 = 2^{k+1} - 1.$$

So, we conclude that this formula is valid for all positive integer values of n.

17. 1. When $n = 1$, $S_1 = 1 = \frac{1(1 + 1)}{2}$.

2. Assume that

$$S_k = 1 + 2 + 3 + 4 + \cdots + k = \frac{k(k + 1)}{2}.$$

Then,

$$S_{k+1} = 1 + 2 + 3 + 4 + \cdots + k + (k + 1) = S_k + (k + 1) = \frac{k(k + 1)}{2} + \frac{2(k + 1)}{2} = \frac{(k + 1)(k + 2)}{2}.$$

So, we conclude that this formula is valid for all positive integer values of n.

19. 1. When $n = 1$, $S_1 = 1^2 = \dfrac{1(2(1) - 1)(2(1) + 1)}{3}$

2. Assume that

$$S_k = 1^2 + 3^2 + \cdots + (2k - 1)^2 = \frac{k(2k - 1)(2k + 1)}{3}$$

Then,

$$\begin{aligned} S_{k+1} &= 1^2 + 3^2 + \cdots + (2k - 1)^2 + (2k + 1)^2 \\ &= S_k + (2k + 1)^2 = \frac{k(2k - 1)(2k + 1)}{3} + (2k + 1)^2 \\ &= (2k + 1)\left[\frac{k(2k - 1)}{3} + (2k + 1)\right] = \frac{2k + 1}{3}\left[2k^2 - k + 6k + 3\right] \\ &= \frac{2k + 1}{3}(2k + 3)(k + 1) = \frac{(k + 1)(2(k + 1) - 1)(2(k + 1) + 1)}{3} \end{aligned}$$

So, we conclude that this formula is valid for all positive integer values of n.

49. $\displaystyle\sum_{n=1}^{15} n = \frac{15(15 + 1)}{2} = 120$

51. $\displaystyle\sum_{n=1}^{6} n^2 = \frac{6(6 + 1)\left[2(6) + 1\right]}{6} = 91$

53. $\displaystyle\sum_{n=1}^{5} n^4 = \frac{5(5 + 1)\left[2(5) + 1\right]\left[3(5)^2 + 3(5) - 1\right]}{30} = 979$

55. $$\begin{aligned} \sum_{n=1}^{6}\left(n^2 - n\right) &= \sum_{n=1}^{6} n^2 - \sum_{n=1}^{6} n \\ &= \frac{6(6 + 1)\left[2(6) + 1\right]}{6} - \frac{6(6 + 1)}{2} \\ &= 91 - 21 = 70 \end{aligned}$$

57. $\displaystyle\sum_{i=1}^{6}\left(6i - 8i^3\right) = 6\sum_{i=1}^{6} i - 8\sum_{i=1}^{6} i^3 = 6\left[\frac{6(6 + 1)}{2}\right] - 8\left[\frac{(6)^2(6 + 1)^2}{4}\right] = 6(21) - 8(441) = -3402$