6. $7, 21, 63, 189, \ldots$

Geometric sequence, $r = 3$

9. $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$

Geometric sequence, $r = -\frac{1}{2}$

12. $9, -6, 4, -\frac{8}{3}, \ldots$

Geometric sequence, $r = -\frac{2}{3}$

15. $1, -\sqrt{7}, 7, -7\sqrt{7}, \ldots$

Geometric sequence, $r = -\sqrt{7}$

18. $a_1 = 8, r = 2$

$$a_1 = 8$$
$$a_2 = 8(2) = 16$$
$$a_3 = 16(2) = 32$$
$$a_4 = 32(2) = 64$$
$$a_5 = 64(2) = 128$$

21. $a_1 = 5, r = -\frac{1}{10}$

$$a_1 = 5$$
$$a_2 = 5\left(-\frac{1}{10}\right) = -\frac{1}{2}$$
$$a_3 = \left(-\frac{1}{2}\right)\left(-\frac{1}{10}\right) = \frac{1}{20}$$
$$a_4 = \frac{1}{20}\left(-\frac{1}{10}\right) = -\frac{1}{200}$$
$$a_5 = \left(-\frac{1}{200}\right)\left(-\frac{1}{10}\right) = \frac{1}{2000}$$

24. $a_1 = 2, r = \pi$

$$a_1 = 2$$
$$a_2 = 2(\pi) = 2\pi$$
$$a_3 = 2\pi(\pi) = 2\pi^2$$
$$a_4 = 2\pi^2(\pi) = 2\pi^3$$
$$a_5 = 2\pi^3(\pi) = 2\pi^4$$

27. $a_1 = 2,\ r = \dfrac{x}{4}$

$$a_1 = 2$$
$$a_2 = 2\left(\frac{x}{4}\right) = \frac{x}{2}$$
$$a_3 = \left(\frac{x}{2}\right)\left(\frac{x}{4}\right) = \frac{x^2}{8}$$
$$a_4 = \left(\frac{x^2}{8}\right)\left(\frac{x}{4}\right) = \frac{x^3}{32}$$
$$a_5 = \left(\frac{x^3}{32}\right)\left(\frac{x}{4}\right) = \frac{x^4}{128}$$

30. $a_1 = 81,\ a_{k+1} = \frac{1}{3}a_k$

$$a_1 = 81$$
$$a_2 = \tfrac{1}{3}(81) = 27$$
$$a_3 = \tfrac{1}{3}(27) = 9$$
$$a_4 = \tfrac{1}{3}(9) = 3$$
$$a_5 = \tfrac{1}{3}(3) = 1$$
$$r = \tfrac{1}{3}$$
$$a_n = 81\left(\tfrac{1}{3}\right)^{n-1} = 243\left(\tfrac{1}{3}\right)^{n}$$

33. $a_1 = 6,\ a_{k+1} = -\frac{3}{2}a_k$

$$a_1 = 6$$
$$a_2 = -\tfrac{3}{2}(6) = -9$$
$$a_3 = -\tfrac{3}{2}(-9) = \tfrac{27}{2}$$
$$a_4 = -\tfrac{3}{2}\left(\tfrac{27}{2}\right) = -\tfrac{81}{4}$$
$$a_5 = -\tfrac{3}{2}\left(-\tfrac{81}{4}\right) = \tfrac{243}{8}$$
$$r = -\tfrac{3}{2}$$
$$a_n = 6\left(-\tfrac{3}{2}\right)^{n-1} \quad \text{or} \quad a_n = -4\left(-\tfrac{3}{2}\right)^{n}$$

36. $a_1 = 5, r = \frac{7}{2}, n = 8$

$$a_n = 5\left(\frac{7}{2}\right)^{n-1}$$

$$a_8 = 5\left(\frac{7}{2}\right)^{8-1} = \frac{4{,}117{,}715}{128}$$

39. $a_1 = 100, r = e^x, n = 9$

$$a_n = a_1 r^{n-1} = 100\left(e^x\right)^{n-1}$$

$$a_9 = 100\left(e^x\right)^8 = 100e^{8x}$$

42. $a_1 = 1, r = \sqrt{3}, n = 8$

$$a_n = a_1 r^{n-1} = \left(\sqrt{3}\right)^{n-1}$$

$$a_8 = \left(\sqrt{3}\right)^7 = 27\sqrt{3}$$

45. $11, 33, 99, \ldots \Rightarrow r = 3$

$$a_n = 11(3)^{n-1}$$

$$a_9 = 11(3)^{9-1} = 72{,}171$$

48. $a_1 = 4, a_2 = 8, a_3 = 16$

$$r = \frac{a_2}{a_1} = \frac{8}{4} = 2$$

$$a_n = a_1 r^{n-1}$$

$$a_{22} = (4)(2)^{21} = 8{,}388{,}608$$

51. $a_1 = 16, a_4 = \frac{27}{4}$

$$a_4 = a_1 r^3$$

$$\frac{27}{4} = 16r^3$$

$$\frac{27}{64} = r^3$$

$$\frac{3}{4} = r$$

$$a_n = 16\left(\frac{3}{4}\right)^{n-1}$$

$$a_3 = 16\left(\frac{3}{4}\right)^2 = 9$$

54. $a_3 = \frac{16}{3}, a_5 = \frac{64}{27}$

$a_5 = a_3 r^{(5-3)}$

$a_5 = a_3 r^2$

$\frac{64}{27} = \frac{16}{3} r^2$

$r^2 = \frac{4}{9}$

$r = \frac{2}{3}$

$a_7 = a_5 r^{(7-5)}$

$a_7 = a_5 r^2$

$a_7 = \left(\frac{64}{27}\right)\left(\frac{2}{3}\right)^2 = \frac{256}{243}$

57. $a_n = 18\left(\frac{2}{3}\right)^{n-1}$

$a_1 = 18$ and $r = \frac{2}{3}$

Because $0 < r < 1$, the sequence is decreasing.

Matches (a).

69. $\sum_{n=1}^{6} (-7)^{n-1} = 1 + (-7) + (-7)^2 + \cdots + (-7)^5 \Rightarrow a_1 = 1, r = -7$

$$S_6 = \frac{1\left(1 - (-7)^6\right)}{1 - (-7)} = -14{,}706$$

72. $\sum_{i=1}^{10} 2\left(\frac{1}{4}\right)^{i-1} = 2 + 2\left(\frac{1}{4}\right)^1 + 2\left(\frac{1}{4}\right)^2 + \cdots + 2\left(\frac{1}{4}\right)^9 \Rightarrow a_1 = 2, r = \frac{1}{4}$

$$S_{10} = 2\left[\frac{1 - \left(\frac{1}{4}\right)^{10}}{1 - \left(\frac{1}{4}\right)}\right] = \frac{8}{3}\left[1 - \left(\frac{1}{4}\right)^{10}\right] \approx 2.667$$

75. $\sum_{n=0}^{20} 3\left(\frac{3}{2}\right)^n = \sum_{n=1}^{21} 3\left(\frac{3}{2}\right)^{n-1} = 3 + 3\left(\frac{3}{2}\right)^1 + 3\left(\frac{3}{2}\right)^2 + \cdots + 3\left(\frac{3}{2}\right)^{20} \Rightarrow a_1 = 3, r = \frac{3}{2}$

$$S_{21} = 3\left[\frac{1 - \left(\frac{3}{2}\right)^{21}}{1 - \frac{3}{2}}\right] = -6\left[1 - \left(\frac{3}{2}\right)^{21}\right] \approx 29{,}921.311$$

78. $\sum_{n=0}^{20} 10\left(\frac{1}{5}\right)^n = 10 + \sum_{n=1}^{20} 10\left(\frac{1}{5}\right)^n = 10 + \left[10\left(\frac{1}{5}\right)^1 + 10\left(\frac{1}{5}\right)^2 + 10\left(\frac{1}{5}\right)^3 + \cdots + 10\left(\frac{1}{5}\right)^{20}\right] \Rightarrow a_1 = 2, r = \frac{1}{5}$

$$S_{21} = 10 + 2\left[\frac{1 - \left(\frac{1}{5}\right)^{20}}{1 - \left(\frac{1}{5}\right)}\right] = 10 + \frac{5}{2}\left[1 - \left(\frac{1}{5}\right)^{20}\right] \approx 12.500$$

81. $\sum_{n=0}^{40} 2\left(-\frac{1}{4}\right)^n = 2 + 2\left(-\frac{1}{4}\right) + 2\left(-\frac{1}{4}\right)^2 + \cdots + 2\left(-\frac{1}{4}\right)^{40} \Rightarrow a_1 = 2, r = -\frac{1}{4}, n = 41$

$$S_{41} = 2\left[\frac{1 - \left(-\frac{1}{4}\right)^{41}}{1 - \left(-\frac{1}{4}\right)}\right] = \frac{8}{5}\left[1 - \left(-\frac{1}{4}\right)^{41}\right] \approx 1.6 = \frac{8}{5}$$

84. $\sum_{i=0}^{25} 8\left(-\frac{1}{2}\right)^i = 8 + \sum_{i=1}^{25} 8\left(-\frac{1}{2}\right)^i = 8 + \left[-4 + 8\left(-\frac{1}{2}\right)^2 + 8\left(-\frac{1}{2}\right)^3 + \cdots + 8\left(-\frac{1}{2}\right)^{25}\right] \Rightarrow a_1 = -4, r = -\frac{1}{2}$

$$S_{26} = 8 - 4\left[\frac{1 - \left(-\frac{1}{2}\right)^{25}}{1 - \left(-\frac{1}{2}\right)}\right] = 8 - \frac{8}{3}\left[1 - \left(-\frac{1}{2}\right)^{25}\right] \approx 5.333$$

87. $10 + 30 + 90 + \cdots + 7290$

$r = 3$ and $7290 = 10(3)^{n-1}$

$729 = 3^{n-1}$

$6 = n - 1 \Rightarrow n = 7$

So, the sum can be written as $\sum_{n=1}^{7} 10(3)^{n-1}$.

90. $15 - 3 + \frac{3}{5} - \cdots - \frac{3}{625}$

$a_1 = 15, r = -\frac{1}{5}$

$$15\left(-\frac{1}{5}\right)^{n-1} = -\frac{3}{625}$$

$$\left(-\frac{1}{5}\right)^{n-1} = -\frac{1}{3125}$$

$$\left(-\frac{1}{5}\right)^{n} = \frac{1}{15,625}$$

By trial and error, we find that $n = 6$.

So, the sum can be written as $\sum_{n=1}^{6} 15\left(-\frac{1}{5}\right)^{n-1}$.

93. $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = 1 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \cdots$

$a_1 = 1, r = \frac{1}{2}$

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{a_1}{1 - r} = \frac{1}{1 - \left(\frac{1}{2}\right)} = 2$$

96. $\sum_{n=0}^{\infty} 2\left(-\frac{2}{3}\right)^n = 2 + 2\left(-\frac{2}{3}\right)^1 + 2\left(-\frac{2}{3}\right)^2 + \cdots$

$a_1 = 2, r = -\frac{2}{3}$

$$\sum_{n=0}^{\infty} 2\left(-\frac{2}{3}\right)^n = \frac{a_1}{1 - r} = \frac{2}{1 - \left(-\frac{2}{3}\right)} = \frac{6}{5}$$

99. $\sum_{n=0}^{\infty} (0.4)^n = 1 + (0.4)^1 + (0.4)^2 + \cdots$

$a_1 = 1, r = 0.4$

$$\sum_{n=0}^{\infty} (0.4)^n = \frac{1}{1 - 0.4} = \frac{5}{3}$$

102. $\sum_{n=0}^{\infty}\left[-10(0.2)^n\right] = -10 - 10(0.2)^1 - 10(0.2)^2 - \cdots$

$a_1 = -10, r = 0.2$

$$\sum_{n=0}^{\infty} -10(0.2)^n = \frac{-10}{1 - 0.2} = -12.5$$

105. $\frac{1}{9} - \frac{1}{3} + 1 - 3 + \cdots = \sum_{n=0}^{\infty} \frac{1}{9}(-3)^n$

The sum is undefined because

$|r| = |-3| = 3 > 1.$

117. $A = \sum_{n=1}^{60} 100\left(1 + \frac{0.06}{12}\right)^n = \sum_{n=1}^{60} 100(1.005)^n = 100(1.005) \cdot \frac{\left[1 - 1.005^{60}\right]}{\left[1 - 1.005\right]} \approx \$7011.89$

133. $a_n = 45{,}000(1.05)^{n-1}$

Total compensation $= T = \sum_{n=1}^{40} 45{,}000(1.05)^{n-1} = 45{,}000\frac{\left(1 - 1.05^{40}\right)}{\left(1 - 1.05\right)} \approx \$5{,}435{,}989.84$