1. infinite sequence

2. terms

3. finite

4. recursively

5. factorial

6. summation notation

7. index; upper; lower

8. series

9. $a_n = 2n + 5$

$a_1 = 2(1) + 5 = 7$

$a_2 = 2(2) + 5 = 9$

$a_3 = 2(3) + 5 = 11$

$a_4 = 2(4) + 5 = 13$

$a_5 = 2(5) + 5 = 15$

10. $a_n = 4n - 7$

$a_1 = 4(1) - 7 = -3$

$a_2 = 4(2) - 7 = 1$

$a_3 = 4(3) - 7 = 5$

$a_4 = 4(4) - 7 = 9$

$a_5 = 4(5) - 7 = 13$

11. $a_n = 2^n$

$a_1 = 2^1 = 2$

$a_2 = 2^2 = 4$

$a_3 = 2^3 = 8$

$a_4 = 2^4 = 16$

$a_5 = 2^5 = 32$

23. $a_n = \dfrac{1}{n^{3/2}}$

$a_1 = \dfrac{1}{1} = 1$

$a_2 = \dfrac{1}{2^{3/2}}$

$a_3 = \dfrac{1}{3^{3/2}}$

$a_4 = \dfrac{1}{4^{3/2}} = \dfrac{1}{8}$

$a_5 = \dfrac{1}{5^{3/2}}$

25. $a_n = \dfrac{(-1)^n}{n^2}$

$a_1 = -\dfrac{1}{1} = -1$

$a_2 = \dfrac{1}{4}$

$a_3 = -\dfrac{1}{9}$

$a_4 = \dfrac{1}{16}$

$a_5 = -\dfrac{1}{25}$

27. $a_n = \frac{2}{3}$

$a_1 = \frac{2}{3}$

$a_2 = \frac{2}{3}$

$a_3 = \frac{2}{3}$

$a_4 = \frac{2}{3}$

$a_5 = \frac{2}{3}$

29. $a_n = n(n-1)(n-2)$

$a_1 = (1)(0)(-1) = 0$

$a_2 = (2)(1)(0) = 0$

$a_3 = (3)(2)(1) = 6$

$a_4 = (4)(3)(2) = 24$

$a_5 = (5)(4)(3) = 60$

31. $a_n = \dfrac{(-1)^{n+1}}{n^2+1}$

$a_1 = \dfrac{(-1)^{1+1}}{1^2+1} = \dfrac{(-1)^2}{2} = \dfrac{1}{2}$

$a_2 = \dfrac{(-1)^{2+1}}{2^2+1} = \dfrac{(-1)^3}{5} = -\dfrac{1}{5}$

$a_3 = \dfrac{(-1)^{3+1}}{3^2+1} = \dfrac{(-1)^4}{10} = \dfrac{1}{10}$

$a_4 = \dfrac{(-1)^{4+1}}{4^2+1} = \dfrac{(-1)^5}{17} = -\dfrac{1}{17}$

$a_5 = \dfrac{(-1)^{5+1}}{5^2+1} = \dfrac{(-1)^6}{26} = \dfrac{1}{26}$

33. $a_{25} = (-1)^{25}(3(25)-2) = -73$

35. $a_{11} = \dfrac{4(11)}{2(11)^2 - 3} = \dfrac{44}{239}$

43. $a_n = \dfrac{8}{n+1}$

$a_1 = 4,\ a_{10} = \dfrac{8}{11}$

The sequence decreases.

Matches graph (c).

44. $a_n = \dfrac{8n}{n+1}$

$a_1 = 4,\ a_3 = \dfrac{24}{4} = 6$

The sequence increases.

Matches graph (b).

45. $a_n = 4(0.5)^{n-1}$

$a_1 = 4,\ a_{10} = \dfrac{1}{128}$

The sequence decreases.

Matches graph (d).

46. $a_n = \dfrac{4^n}{n!}$

$a_1 = 4,\ a_4 = \dfrac{4^4}{4!} = \dfrac{256}{24} = 10\dfrac{2}{3}$

The sequence increases.

Matches graph (a).

51. $-\dfrac{2}{3}, \dfrac{3}{4}, -\dfrac{4}{5}, \dfrac{5}{6}, -\dfrac{6}{7}, \ldots$

$a_n = (-1)^n\left(\dfrac{n+1}{n+2}\right)$

53. $\dfrac{2}{1}, \dfrac{3}{3}, \dfrac{4}{5}, \dfrac{5}{7}, \dfrac{6}{9}, \ldots$

$a_n = \dfrac{n+1}{2n-1}$

55. $1, \dfrac{1}{4}, \dfrac{1}{9}, \dfrac{1}{16}, \dfrac{1}{25}, \ldots$

$a_n = \dfrac{1}{n^2}$

57. $1, -1, 1, -1, 1, \ldots$

n:	1	2	3	4	5	...	n
Terms:	1	-1	1	-1	1	...	a_n

Apparent pattern:

Each term is either 1 or -1 which implies that $a_n = (-1)^{n+1}$.

63. $a_1 = 28$ and $a_{k+1} = a_k - 4$

$a_1 = 28$

$a_2 = a_1 - 4 = 28 - 4 = 24$

$a_3 = a_2 - 4 = 24 - 4 = 20$

$a_4 = a_3 - 4 = 20 - 4 = 16$

$a_5 = a_4 - 4 = 16 - 4 = 12$

65. $a_1 = 3$, and $a_{k+1} = 2(a_k - 1)$

$a_1 = 3$

$a_2 = 2(a_1 - 1) = 2(3 - 1) = 4$

$a_3 = 2(a_2 - 1) = 2(4 - 1) = 6$

$a_4 = 2(a_3 - 1) = 2(6 - 1) = 10$

$a_5 = 2(a_4 - 1) = 2(10 - 1) = 18$

67. $a_1 = 6$ and $a_{k+1} = a_k + 2$

$a_1 = 6$

$a_2 = a_1 + 2 = 6 + 2 = 8$

$a_3 = a_2 + 2 = 8 + 2 = 10$

$a_4 = a_3 + 2 = 10 + 2 = 12$

$a_5 = a_4 + 2 = 12 + 2 = 14$

In general, $a_n = 2n + 4$.

69. $a_1 = 81$ and $a_{k+1} = \frac{1}{3}a_k$

$a_1 = 81$

$a_2 = \frac{1}{3}a_1 = \frac{1}{3}(81) = 27$

$a_3 = \frac{1}{3}a_2 = \frac{1}{3}(27) = 9$

$a_4 = \frac{1}{3}a_3 = \frac{1}{3}(9) = 3$

$a_5 = \frac{1}{3}a_4 = \frac{1}{3}(3) = 1$

In general,

$$a_n = 81\left(\frac{1}{3}\right)^{n-1} = 81(3)\left(\frac{1}{3}\right)^n = \frac{243}{3^n}.$$

71. $a_n = \dfrac{1}{n!}$

$a_0 = \dfrac{1}{0!} = \dfrac{1}{1} = 1$

$a_1 = \dfrac{1}{1!} = \dfrac{1}{1} = 1$

$a_2 = \dfrac{1}{2!} = \dfrac{1}{2 \cdot 1} = \dfrac{1}{2}$

$a_3 = \dfrac{1}{3!} = \dfrac{1}{3 \cdot 2 \cdot 1} = \dfrac{1}{6}$

$a_4 = \dfrac{1}{4!} = \dfrac{1}{4 \cdot 3 \cdot 2 \cdot 1} = \dfrac{1}{24}$

73. $a_n = \dfrac{1}{(n+1)!}$

$a_0 = \dfrac{1}{1!} = 1$

$a_1 = \dfrac{1}{2!} = \dfrac{1}{2}$

$a_2 = \dfrac{1}{3!} = \dfrac{1}{6}$

$a_3 = \dfrac{1}{4!} = \dfrac{1}{24}$

$a_4 = \dfrac{1}{5!} = \dfrac{1}{120}$

77. $\dfrac{4!}{6!} = \dfrac{\cancel{1 \cdot 2 \cdot 3 \cdot 4}}{\cancel{1 \cdot 2 \cdot 3 \cdot 4} \cdot 5 \cdot 6} = \dfrac{1}{5 \cdot 6} = \dfrac{1}{30}$

79. $\dfrac{12!}{4! \cdot 8!} = \dfrac{\cancel{1 \cdot 2 \cdot 3 \cdot 4} \cdot \cancel{5 \cdot 6 \cdot 7 \cdot 8} \cdot 9 \cdot 10 \cdot 11 \cdot 12}{\cancel{1 \cdot 2 \cdot 3 \cdot 4} \cdot 1 \cdot 2 \cdot 3 \cdot 4 \cdot \cancel{5 \cdot 6 \cdot 7 \cdot 8}} = \dfrac{9 \cdot \overset{5}{\cancel{10}} \cdot 11 \cdot \cancel{12}}{1 \cdot \cancel{2} \cdot \cancel{3 \cdot 4}} = 495$

85. $\displaystyle\sum_{i=1}^{5} (2i + 1) = (2 + 1) + (4 + 1) + (6 + 1) + (8 + 1) + (10 + 1) = 35$

87. $\displaystyle\sum_{k=1}^{4} 10 = 10 + 10 + 10 + 10 = 40$

89. $\displaystyle\sum_{i=0}^{4} i^2 = 0^2 + 1^2 + 2^2 + 3^2 + 4^2 = 30$

91. $\sum_{k=0}^{3} \frac{1}{k^2 + 1} = \frac{1}{1} + \frac{1}{1+1} + \frac{1}{4+1} + \frac{1}{9+1} = \frac{9}{5}$

93. $\sum_{k=2}^{5} (k+1)^2(k-3) = (3)^2(-1) + (4)^2(0) + (5)^2(1) + (6)^2(2) = 88$

95. $\sum_{i=1}^{4} 2^i = 2^1 + 2^2 + 2^3 + 2^4 = 30$

103. $\frac{1}{3(1)} + \frac{1}{3(2)} + \frac{1}{3(3)} + \cdots + \frac{1}{3(9)} = \sum_{i=1}^{9} \frac{1}{3i}$

105. $\left[2\left(\frac{1}{8}\right) + 3\right] + \left[2\left(\frac{2}{8}\right) + 3\right] + \left[2\left(\frac{3}{8}\right) + 3\right] + \cdots + \left[2\left(\frac{8}{8}\right) + 3\right] = \sum_{i=1}^{8}\left[2\left(\frac{i}{8}\right) + 3\right]$

107. $3 - 9 + 27 - 81 + 243 - 729 = \sum_{i=1}^{6} (-1)^{i+1} 3i$

109. $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots - \frac{1}{20^2} = \sum_{i=1}^{20} \frac{(-1)^{i+1}}{i^2}$