1. $a_n = 2 + \frac{6}{n}$

$a_1 = 2 + \frac{6}{1} = 8$

$a_2 = 2 + \frac{6}{2} = 5$

$a_3 = 2 + \frac{6}{3} = 4$

$a_4 = 2 + \frac{6}{4} = \frac{7}{2}$

$a_5 = 2 + \frac{6}{5} = \frac{16}{5}$

3. $a_n = \frac{72}{n!}$

$a_1 = \frac{72}{1!} = 72$

$a_2 = \frac{72}{2!} = 36$

$a_3 = \frac{72}{3!} = 12$

$a_4 = \frac{72}{4!} = 3$

$a_5 = \frac{72}{5!} = \frac{3}{5}$

5. $-2, 2, -2, 2, -2, \ldots$

$a_n = 2(-1)^n$

7. $4, 2, \frac{4}{3}, 1, \frac{4}{5}, \ldots$

$a_n = \frac{4}{n}$

9. $9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362{,}880$

11. $\frac{3!\,5!}{6!} = \frac{(3 \cdot 2 \cdot 1)5!}{6 \cdot 5!} = 1$

13. $\sum_{i=1}^{6} 8 = 8 + 8 + 8 + 8 + 8 + 8 = 48$

15. $\sum_{j=1}^{4} \frac{6}{j^2} = \frac{6}{1^2} + \frac{6}{2^2} + \frac{6}{3^2} + \frac{6}{4^2}$

$= 6 + \frac{3}{2} + \frac{2}{3} + \frac{3}{8}$

$= \frac{205}{24}$

25. $6, -1, -8, -15, -22, \ldots$

Arithmetic sequence, $d = -7$

27. $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$

Arithmetic sequence, $d = \frac{1}{2}$

29. $a_1 = 3, d = 11$

$a_1 = 3$

$a_2 = 3 + 11 = 14$

$a_3 = 14 + 11 = 25$

$a_4 = 25 + 11 = 36$

$a_5 = 36 + 11 = 47$

31. $a_1 = 25, a_{k+1} = a_k + 3$

$a_1 = 25$

$a_2 = 25 + 3 = 28$

$a_3 = 28 + 3 = 31$

$a_4 = 31 + 3 = 34$

$a_5 = 34 + 3 = 37$

33. $a_1 = 7, d = 12$

$a_n = 7 + (n - 1)12$

$= 7 + 12n - 12$

$= 12n - 5$

35. $a_1 = y, d = 3y$

$a_n = y + (n - 1)3y$

$= y + 3ny - 3y$

$= 3ny - 2y$

37. $a_2 = 93, a_6 = 65$

$a_6 = a_2 + 4d \Rightarrow 65 = 93 + 4d \Rightarrow -28 = 4d \Rightarrow d = -7$

$a_1 = a_2 - d \Rightarrow a_1 = 93 - (-7) = 100$

$a_n = a_1 + (n-1)d = 100 + (n-1)(-7) = -7n + 107$

39. $\sum_{k=1}^{100} 7k$ is arithmetic. Therefore, $a_1 = 7, a_{100} = 700, S_{700} = \frac{100}{2}(7 + 700) = 35{,}350.$

41. $\sum_{j=1}^{10} (2j - 3)$ is arithmetic. Therefore, $a_1 = -1, a_{10} = 17, S_{10} = \frac{10}{2}[-1 + 17] = 80.$

47. $6, 12, 24, 48, \ldots$

Geometric sequence, $r = 2$

49. $\frac{1}{5}, -\frac{3}{5}, \frac{9}{5}, -\frac{27}{5}, \ldots$

Geometric sequence, $r = -3$

51. $a_1 = 4, r = -\frac{1}{4}$

$a_1 = 4$

$a_2 = 4\left(-\frac{1}{4}\right) = -1$

$a_3 = -1\left(-\frac{1}{4}\right) = \frac{1}{4}$

$a_4 = \frac{1}{4}\left(-\frac{1}{4}\right) = -\frac{1}{16}$

$a_5 = -\frac{1}{16}\left(-\frac{1}{4}\right) = \frac{1}{64}$

53. $a_1 = 9, a_3 = 4$

$a_3 = a_1 r^2$

$4 = 9r^2$

$\frac{4}{9} = r^2 \Rightarrow r = \pm\frac{2}{3}$

$a_1 = 9$		$a_1 = 9$
$a_2 = 9\left(\frac{2}{3}\right) = 6$		$a_2 = 9\left(-\frac{2}{3}\right) = -6$
$a_3 = 6\left(\frac{2}{3}\right) = 4$	or	$a_3 = -6\left(-\frac{2}{3}\right) = 4$
$a_4 = 4\left(\frac{2}{3}\right) = \frac{8}{3}$		$a_4 = 4\left(-\frac{2}{3}\right) = -\frac{8}{3}$
$a_5 = \frac{8}{3}\left(\frac{2}{3}\right) = \frac{16}{9}$		$a_5 = -\frac{8}{3}\left(-\frac{2}{3}\right) = \frac{16}{9}$

55. $a_1 = 18, a_2 = -9$

$$a_2 = a_1 r$$
$$-9 = 18r$$
$$-\tfrac{1}{2} = r$$
$$a_n = 18\left(-\tfrac{1}{2}\right)^{n-1}$$
$$a_{10} = 18\left(-\tfrac{1}{2}\right)^{9} = \tfrac{9}{256}$$

57. $a_1 = 100, r = 1.05$

$$a_n = 100(1.05)^{n-1}$$
$$a_{10} = 100(1.05)^{9} \approx 155.133$$

59. $\displaystyle\sum_{i=1}^{7} 2^{i-1} = \frac{1-2^7}{1-2} = 127$

67. $\displaystyle\sum_{i=1}^{\infty} \left(\frac{7}{8}\right)^{i-1} = \frac{1}{1-\frac{7}{8}} = 8$

69. $\displaystyle\sum_{k=1}^{\infty} 4\left(\frac{2}{3}\right)^{k-1} = \frac{4}{1-\frac{2}{3}} = 12$

73. 1. When $n = 1, 3 = 1(1 + 2)$.

2. Assume that $S_k = 3 + 5 + 7 + \cdots + (2k + 1) = k(k + 2)$.

Then, $S_{k+1} = 3 + 5 + 7 + \cdots + (2k + 1) + [2(k + 1) + 1] = S_k + (2k + 3)$

$$= k(k + 2) + 2k + 3$$
$$= k^2 + 4k + 3$$
$$= (k + 1)(k + 3)$$
$$= (k + 1)[(k + 1) + 2].$$

So, by mathematical induction, the formula is valid for all positive integer values of n.

74. 1. When $n = 1$, $S_1 = 1 = \frac{1}{4}(1 + 3) = 1$.

2. Assume that $S_k = 1 + \frac{3}{2} + 2 + \frac{5}{2} + \cdots + \frac{1}{2}(k + 1) = \frac{k}{4}(k + 3)$. Then,

$$\begin{aligned} S_{k+1} = S_k + a_{k+1} &= \left(1 + \frac{3}{2} + 2 + \frac{5}{2} + \cdots + \frac{1}{2}(k + 1)\right) + \frac{1}{2}(k + 2) \\ &= \frac{k}{4}(k + 3) + \frac{1}{2}(k + 2) \\ &= \frac{k(k + 3) + 2(k + 2)}{4} \\ &= \frac{k^2 + 5k + 4}{4} \\ &= \frac{(k + 1)(k + 4)}{4} \\ &= \frac{k + 1}{4}\left[(k + 1) + 3\right]. \end{aligned}$$

So, the formula holds for all positive integers n.

83. $a_1 = f(1) = 5$, $a_n = a_{n-1} + 5$

$a_1 = 5$

$a_2 = 5 + 5 = 10$

$a_3 = 10 + 5 = 15$

$a_4 = 15 + 5 = 20$

$a_5 = 20 + 5 = 25$

n:	1		2		3		4		5
a_n:	5		10		15		20		25
First differences:		5		5		5		5	
Second differences:			0		0		0		

Because the first differences are all the same, the sequence has a linear model.

84. $a_1 = -3$

$a_n = a_{n-1} - 2n$

$a_1 = -3$

$a_2 = a_1 - 2(2) = -3 - 4 = -7$

$a_3 = a_2 - 2(3) = -7 - 6 = -13$

$a_4 = a_3 - 2(4) = -13 - 8 = -21$

$a_5 = a_4 - 2(5) = -21 - 10 = -31$

$$\begin{array}{rccccccccc}
a_n: & -3 & & -7 & & -13 & & -21 & & -31 \\
\text{First differences:} & & -4 & & -6 & & -8 & & -10 & \\
\text{Second differences:} & & & -2 & & -2 & & -2 & &
\end{array}$$

Because the second differences are all the same, the sequence has a quadratic model.

91. $(x + 4)^4 = x^4 + 4x^3(4) + 6x^2(4)^2 + 4x(4)^3 + 4^4 = x^4 + 16x^3 + 96x^2 + 256x + 256$

93. $(a - 3b)^5 = a^5 - 5a^4(3b) + 10a^3(3b)^2 - 10a^2(3b)^3 + 5a(3b)^4 - (3b)^5$

$= a^5 - 15a^4b + 90a^3b^2 - 270a^2b^3 + 405ab^4 - 243b^5$

97.

First number:	1	2	3	4	5	6	7	8	9	10	11
Second number:	11	10	9	8	7	6	5	4	3	2	1

From this list, you can see that a total of 12 occurs 11 different ways.

98. ${}_6C_1 \cdot {}_5C_1 \cdot {}_6C_1 = 6 \cdot 5 \cdot 6 = 180$

99. $(10)(10)(10)(10) = 10{,}000$ different telephone numbers

100. ${}_3C_1 \cdot {}_4C_1 \cdot {}_6C_1 = 3 \cdot 4 \cdot 6 = 72$

101. ${}_{10}P_3 = \dfrac{10!}{7!} = \dfrac{10 \cdot 9 \cdot 8 \cdot 7!}{7!}$

$= 10 \cdot 9 \cdot 8 = 720$ different ways

102. ${}_{32}C_{12} = \dfrac{32!}{20!12!} = 225{,}792{,}840$

103. ${}_8C_3 = \dfrac{8!}{5!3!} = 56$

104. Breads: ${}_5C_1 = 5$

Meats: ${}_4C_0 + {}_4C_1 + {}_4C_2 + {}_4C_3 + {}_4C_4 = 1 + 4 + 6 + 4 + 1 = 16$

Cheeses: ${}_3C_0 + {}_3C_1 + {}_3C_2 + {}_3C_3 = 1 + 3 + 3 + 1 = 8$

Vegetables: ${}_6C_0 + {}_6C_1 + {}_6C_2 + {}_6C_3 + {}_6C_4 + {}_6C_5 + {}_6C_6 = 1 + 6 + 15 + 20 + 15 + 6 + 1 = 64$

$5 \cdot 16 \cdot 8 \cdot 64 = 40{,}960$

107. (a) $25\% + 18\% = 43\%$

(b) $100\% - 18\% = 82\%$

108. (a) $\frac{208}{500} = 0.416$ or 41.6%

(b) $\frac{400}{500} = 0.8$ or 80%

(c) $\frac{37}{500} = 0.074$ or 7.4%

109. $\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{1296}$

110. $\left(\frac{6}{6}\right)\left(\frac{5}{6}\right)\left(\frac{4}{6}\right)\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)\left(\frac{1}{6}\right) = \frac{6!}{6^6} = \frac{720}{46{,}656} = \frac{5}{324}$

111. $1 - \frac{13}{52} = 1 - \frac{1}{4} = \frac{3}{4}$

112. $1 - P(HHHHH) = 1 - \left(\frac{1}{2}\right)^5 = \frac{31}{32}$