1. Given: $A = 38°, B = 70°, a = 8$

$$C = 180° - 38° - 70° = 72°$$

$$b = \frac{a \sin B}{\sin A} = \frac{8 \sin 70°}{\sin 38°} \approx 12.21$$

$$c = \frac{a \sin C}{\sin A} = \frac{8 \sin 72°}{\sin 38°} \approx 12.36$$

3. Given: $B = 72°, C = 82°, b = 54$

$$A = 180° - 72° - 82° = 26°$$

$$a = \frac{b \sin A}{\sin B} = \frac{54 \sin 26°}{\sin 72°} \approx 24.89$$

$$c = \frac{b \sin C}{\sin B} = \frac{54 \sin 82°}{\sin 72°} \approx 56.23$$

5. Given: $A = 16°, B = 98°, c = 8.4$

$$C = 180° - 16° - 98° = 66°$$

$$a = \frac{c \sin A}{\sin C} = \frac{8.4 \sin 16°}{\sin 66°} \approx 2.53$$

$$b = \frac{c \sin B}{\sin C} = \frac{8.4 \sin 98°}{\sin 66°} \approx 9.11$$

7. Given: $A = 24°, C = 48°, b = 27.5$

$$B = 180° - 24° - 48° = 108°$$

$$a = \frac{b \sin A}{\sin B} = \frac{27.5 \sin 24°}{\sin 108°} \approx 11.76$$

$$c = \frac{b \sin C}{\sin B} = \frac{27.5 \sin 48°}{\sin 108°} \approx 21.49$$

9. Given: $B = 150°, b = 30, c = 10$

$$\sin C = \frac{c \sin B}{b} = \frac{10 \sin 150°}{30} \approx 0.1667 \Rightarrow C \approx 9.59°$$

$$A \approx 180° - 150° - 9.59° = 20.41°$$

$$a = \frac{b \sin A}{\sin B} = \frac{30 \sin 20.41°}{\sin 150°} \approx 20.92$$

11. $A = 75°, a = 51.2, b = 33.7$

$$\sin B = \frac{b \sin A}{a} = \frac{33.7 \sin 75°}{51.2} \approx 0.6358 \Rightarrow B \approx 39.48°$$

$$C \approx 180° - 75° - 39.48° = 65.52°$$

$$c = \frac{a \sin C}{\sin A} = \frac{51.2 \sin 65.52°}{\sin 75°} \approx 48.24$$

13. $A = 33°, b = 7, c = 10$

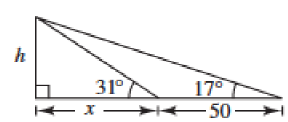
$$\text{Area} = \tfrac{1}{2}bc \sin A = \tfrac{1}{2}(7)(10) \sin 33° \approx 19.06$$

15. $C = 119°, a = 18, b = 6$

$$\text{Area} = \tfrac{1}{2}ab \sin C = \tfrac{1}{2}(18)(6) \sin 119° \approx 47.23$$

17. $\tan 17° = \dfrac{h}{x + 50} \Rightarrow h = (x + 50) \tan 17°$

$$h = x \tan 17° + 50 \tan 17°$$



$$\tan 31° = \frac{h}{x} \Rightarrow h = x \tan 31°$$

$$x \tan 17° + 50 \tan 17° = x \tan 31°$$

$$50 \tan 17° = x(\tan 31° - \tan 17°)$$

$$\frac{50 \tan 17°}{\tan 31° - \tan 17°} = x$$

$$x \approx 51.7959$$

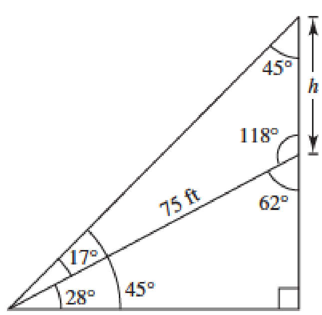
$$h = x \tan 31° \approx 51.7959 \tan 31° \approx 31.1 \text{ meters}$$

The height of the building is approximately 31.1 meters.

19. $\dfrac{h}{\sin 17°} = \dfrac{75}{\sin 45°}$

$$h = \frac{75 \sin 17°}{\sin 45°}$$

$$h \approx 31.01 \text{ feet}$$



21. Given: $a = 8, b = 14, c = 17$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{64 + 196 - 289}{2(8)(14)} \approx -0.1295 \Rightarrow C \approx 97.44°$$

$$\sin B = \frac{b \sin C}{c} \approx \frac{14 \sin 97.44}{17} \approx 0.8166 \Rightarrow B \approx 54.75°$$

$$A \approx 180° - 54.75° - 97.44° = 27.81°$$

23. Given: $a = 6, b = 9, c = 14$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{36 + 81 - 196}{2(6)(9)} \approx -0.7315 \Rightarrow C \approx 137.01°$$

$$\sin B = \frac{b \sin C}{c} \approx \frac{9 \sin 137.01°}{14} \approx 0.4383 \Rightarrow B \approx 26.00°$$

$$A \approx 180° - 26.00° - 137.01° = 16.99°$$

25. Given: $a = 2.5, b = 5.0, c = 4.5$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = 0.0667 \Rightarrow B \approx 86.18°$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = 0.44 \Rightarrow C \approx 63.90°$$

$$A = 180° - B - C \approx 29.92°$$

27. Given: $B = 108°, a = 11, c = 11$

$$b^2 = a^2 + c^2 - 2ac \cos B = 11^2 + 11^2 - 2(11)(11) \cos 108° \Rightarrow b \approx 17.80$$

$$A = C = \tfrac{1}{2}(180° - 108°) = 36°$$

29. Given: $C = 43°, a = 22.5, b = 31.4$

$$c = \sqrt{a^2 + b^2 - 2ab \cos C} \approx 21.42$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx -0.02169 \Rightarrow B \approx 91.24°$$

$$A = 180° - B - C \approx 45.76°$$

39. $a = 3, b = 6, c = 8$

$$s = \frac{a + b + c}{2} = \frac{3 + 6 + 8}{2} = 8.5$$

$$\begin{aligned} \text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{8.5(5.5)(2.5)(0.5)} \\ &\approx 7.64 \end{aligned}$$

45. Initial point: $(-5, 4)$

Terminal point: $(2, -1)$

$\mathbf{v} = \langle 2 - (-5), -1 - 4 \rangle = \langle 7, -5 \rangle$

47. Initial point: $(0, 10)$

Terminal point: $(7, 3)$

$\mathbf{v} = \langle 7 - 0, 3 - 10 \rangle = \langle 7, -7 \rangle$

49. $\|\mathbf{v}\| = 8, \theta = 120°$

$\langle 8 \cos 120°, 8 \sin 120° \rangle = \langle -4, 4\sqrt{3} \rangle$

51. $\mathbf{u} = \langle -1, -3 \rangle, \mathbf{v} = \langle -3, 6 \rangle$

(a) $\mathbf{u} + \mathbf{v} = \langle -1, -3 \rangle + \langle -3, 6 \rangle = \langle -4, 3 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -1, -3 \rangle - \langle -3, 6 \rangle = \langle 2, -9 \rangle$

(c) $4\mathbf{u} = 4\langle -1, -3 \rangle = \langle -4, -12 \rangle$

(d) $3\mathbf{v} + 5\mathbf{u} = 3\langle -3, 6 \rangle + 5\langle -1, -3 \rangle = \langle -9, 18 \rangle + \langle -5, -15 \rangle = \langle -14, 3 \rangle$

53. $\mathbf{u} = \langle -5, 2 \rangle, \mathbf{v} = \langle 4, 4 \rangle$

(a) $\mathbf{u} + \mathbf{v} = \langle -5, 2 \rangle + \langle 4, 4 \rangle = \langle -1, 6 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -5, 2 \rangle - \langle 4, 4 \rangle = \langle -9, -2 \rangle$

(c) $4\mathbf{u} = 4\langle -5, 2 \rangle = \langle -20, 8 \rangle$

(d) $3\mathbf{v} + 5\mathbf{u} = 3\langle 4, 4 \rangle + 5\langle -5, 2 \rangle = \langle 12, 12 \rangle + \langle -25, 10 \rangle = \langle -13, 22 \rangle$

55. $\mathbf{u} = 2\mathbf{i} - \mathbf{j}, \mathbf{v} = 5\mathbf{i} + 3\mathbf{j}$

(a) $\mathbf{u} + \mathbf{v} = (2\mathbf{i} - \mathbf{j}) + (5\mathbf{i} + 3\mathbf{j}) = 7\mathbf{i} + 2\mathbf{j}$

(b) $\mathbf{u} - \mathbf{v} = (2\mathbf{i} - \mathbf{j}) - (5\mathbf{i} + 3\mathbf{j}) = -3\mathbf{i} - 4\mathbf{j}$

(c) $4\mathbf{u} = 4(2\mathbf{i} - \mathbf{j}) = 8\mathbf{i} - 4\mathbf{j}$

(d) $3\mathbf{v} + 5\mathbf{u} = 3(5\mathbf{i} + 3\mathbf{j}) + 5(2\mathbf{i} - \mathbf{j}) = 15\mathbf{i} + 9\mathbf{j} + 10\mathbf{i} - 5\mathbf{j} = 25\mathbf{i} + 4\mathbf{j}$

57. $\mathbf{u} = 4\mathbf{i}, \mathbf{v} = -\mathbf{i} + 6\mathbf{j}$

(a) $\mathbf{u} + \mathbf{v} = 4\mathbf{i} + (-\mathbf{i} + 6\mathbf{j}) = 3\mathbf{i} + 6\mathbf{j}$

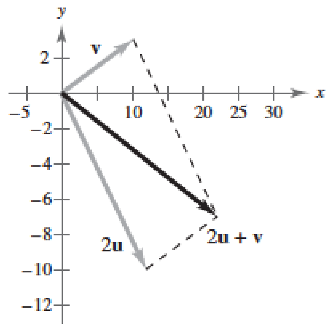
(b) $\mathbf{u} - \mathbf{v} = 4\mathbf{i} - (-\mathbf{i} + 6\mathbf{j}) = 5\mathbf{i} - 6\mathbf{j}$

(c) $4\mathbf{u} = 4(4\mathbf{i}) = 16\mathbf{i}$

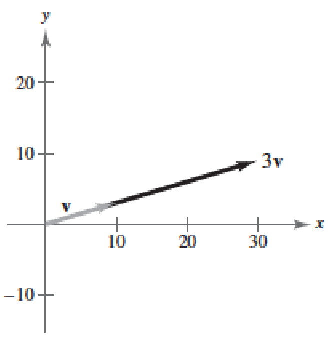
(d) $3\mathbf{v} + 5\mathbf{u} = 3(-\mathbf{i} + 6\mathbf{j}) + 5(4\mathbf{i}) = -3\mathbf{i} + 18\mathbf{j} + 20\mathbf{i} = 17\mathbf{i} + 18\mathbf{j}$

59. $\mathbf{u} = 6\mathbf{i} - 5\mathbf{j}, \mathbf{v} = 10\mathbf{i} + 3\mathbf{j}$

$$\begin{aligned} 2\mathbf{u} + \mathbf{v} &= 2(6\mathbf{i} - 5\mathbf{j}) + (10\mathbf{i} + 3\mathbf{j}) \\ &= 22\mathbf{i} - 7\mathbf{j} \\ &= \langle 22, -7 \rangle \end{aligned}$$



61.
$$\begin{aligned} \mathbf{v} &= 10\mathbf{i} + 3\mathbf{j} \\ 3\mathbf{v} &= 3(10\mathbf{i} + 3\mathbf{j}) \\ &= 30\mathbf{i} + 9\mathbf{j} \\ &= \langle 30, 9 \rangle \end{aligned}$$



63. $\mathbf{u} = \langle -1, 5 \rangle = -\mathbf{i} + 5\mathbf{j}$

65. Initial point: $(3, 4)$

Terminal point: $(9, 8)$

$\mathbf{u} = (9 - 3)\mathbf{i} + (8 - 4)\mathbf{j} = 6\mathbf{i} + 4\mathbf{j}$

67. $\mathbf{v} = -10\mathbf{i} + 10\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{(-10)^2 + (10)^2} = \sqrt{200} = 10\sqrt{2}$

$\tan\theta = \dfrac{10}{-10} = -1 \Rightarrow \theta = 135°$ because $\mathbf{v}$ is in Quadrant II.

$\mathbf{v} = 10\sqrt{2}\langle\mathbf{i}\cos 135° + \mathbf{j}\sin 135°\rangle$

69. $\mathbf{v} = 7(\cos 60°\mathbf{i} + \sin 60°\mathbf{j})$

$\|\mathbf{v}\| = 7$

$\theta = 60°$

71. $\mathbf{v} = 5\mathbf{i} + 4\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{5^2 + 4^2} = \sqrt{41}$

$\tan\theta = \frac{4}{5} \Rightarrow \theta \approx 38.7°$

73. $\mathbf{v} = -3\mathbf{i} - 3\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{(-3)^2 + (-3)^2} = 3\sqrt{2}$

$\tan\theta = \dfrac{-3}{-3} = 1 \Rightarrow \theta = 225°$

75. Magnitude of resultant:

$c = \sqrt{85^2 + 50^2 - 2(85)(50)\cos 165°} \approx 133.92$ pounds

Let θ be the angle between the resultant and the 85-pound force.

$\cos\theta \approx \dfrac{(133.92)^2 + 85^2 - 50^2}{2(133.92)(85)} \approx 0.9953 \Rightarrow \theta \approx 5.6°$

77. Airspeed: $\mathbf{u} = 430(\cos 45°\mathbf{i} - \sin 45°\mathbf{j}) = 215\sqrt{2}(\mathbf{i} - \mathbf{j})$

Wind: $\mathbf{w} = 35(\cos 60°\mathbf{i} + \sin 60°\mathbf{j}) = \frac{35}{2}(\mathrm{i} + \sqrt{3}\mathbf{j})$

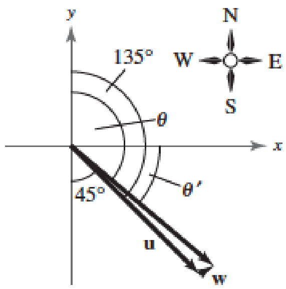
Groundspeed: $\mathbf{u} + \mathbf{w} = \left(215\sqrt{2} + \frac{35}{2}\right)\mathbf{i} + \left(\frac{35\sqrt{3}}{2} - 215\sqrt{2}\right)\mathbf{j}$

$$\|\mathbf{u} + \mathbf{w}\| = \sqrt{\left(215\sqrt{2} + \frac{35}{2}\right)^2 + \left(\frac{35\sqrt{3}}{2} - 215\sqrt{2}\right)^2} \approx 422.30 \text{ miles per hour}$$

Bearing: $\tan\theta' = \dfrac{17.5\sqrt{3} - 215\sqrt{2}}{215\sqrt{2} + 17.5}$

$\theta' \approx -40.4°$

$\theta = 90° + |\theta'| = 130.4°$



79. $\mathbf{u} = \langle 6, 7\rangle, \mathbf{v} = \langle -3, 9\rangle$

$\mathbf{u} \cdot \mathbf{v} = 6(-3) + 7(9) = 45$

81. $\mathbf{u} = 3\mathbf{i} + 7\mathbf{j}, \mathbf{v} = 11\mathbf{i} - 5\mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} = 3(11) + 7(-5) = -2$

83. $\mathbf{u} = \langle -4, 2\rangle$

$2\mathbf{u} = \langle -8, 4\rangle$

$2\mathbf{u} \cdot \mathbf{u} = -8(-4) + 4(2) = 40$

The result is a scalar.

85. $\mathbf{u} = \langle -4, 2\rangle$

$4 - \|\mathbf{u}\| = 4 - \sqrt{(-4)^2 + 2^2} = 4 - \sqrt{20} = 4 - 2\sqrt{5}$

The result is a scalar.

87. $\mathbf{u} = \langle -4, 2 \rangle$, $\mathbf{v} = \langle 5, 1 \rangle$

$$\begin{aligned}\mathbf{u}(\mathbf{u} \cdot \mathbf{v}) &= \langle -4, 2 \rangle[-4(5) + 2(1)] \\ &= -18\langle -4, 2 \rangle \\ &= \langle 72, -36 \rangle\end{aligned}$$

The result is a vector.

89. $\mathbf{u} = \langle -4, 2 \rangle$, $\mathbf{v} = \langle 5, 1 \rangle$

$$\begin{aligned}(\mathbf{u} \cdot \mathbf{u}) - (\mathbf{u} \cdot \mathbf{v}) &= [-4(-4) + 2(2)] - [-4(5) + 2(1)] \\ &= 20 - (-18) \\ &= 38\end{aligned}$$

The result is a scalar.

91. $\mathbf{u} = \cos\dfrac{7\pi}{4}\mathbf{i} + \sin\dfrac{7\pi}{4}\mathbf{j} = \left\langle \dfrac{1}{\sqrt{2}}, -\dfrac{1}{\sqrt{2}} \right\rangle$

$\mathbf{v} = \cos\dfrac{5\pi}{6}\mathbf{i} + \sin\dfrac{5\pi}{6}\mathbf{j} = \left\langle -\dfrac{\sqrt{3}}{2}, \dfrac{1}{2} \right\rangle$

$\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \dfrac{-\sqrt{3} - 1}{2\sqrt{2}} \Rightarrow \theta = \dfrac{11\pi}{12}$

93. $\mathbf{u} = \langle 2\sqrt{2}, -4 \rangle$, $\mathbf{v} = \langle -\sqrt{2}, 1 \rangle$

$\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \dfrac{-8}{\left(\sqrt{24}\right)\left(\sqrt{3}\right)} \Rightarrow \theta \approx 160.5^\circ$

95. $\mathbf{u} = \langle -3, 8 \rangle$

$\mathbf{v} = \langle 8, 3 \rangle$

$\mathbf{u} \cdot \mathbf{v} = -3(8) + 8(3) = 0$

$\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

97. $\mathbf{u} = -\mathbf{i}$

$\mathbf{v} = \mathbf{i} + 2\mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} \neq 0 \Rightarrow$ Not orthogonal

$\mathbf{v} \neq k\mathbf{u} \Rightarrow$ Not parallel

Neither

99. $\mathbf{u} = \langle -4, 3\rangle, \mathbf{v} = \langle -8, -2\rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \left(\frac{26}{68}\right)\langle -8, -2\rangle = -\frac{13}{17}\langle 4, 1\rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle -4, 3\rangle - \left(-\frac{13}{17}\right)\langle 4, 1\rangle = \frac{16}{17}\langle -1, 4\rangle$$

$$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = -\frac{13}{17}\langle 4, 1\rangle + \frac{16}{17}\langle -1, 4\rangle$$

101. $\mathbf{u} = \langle 2, 7\rangle, \mathbf{v} = \langle 1, -1\rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = -\frac{5}{2}\langle 1, -1\rangle = \frac{5}{2}\langle -1, 1\rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 2, 7\rangle - \left(\frac{5}{2}\right)\langle -1, 1\rangle = \frac{9}{2}\langle 1, 1\rangle$$

$$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = \frac{5}{2}\langle -1, 1\rangle + \frac{9}{2}\langle 1, 1\rangle$$